



South Australian
Certificate of Education

Mathematical Methods 2023

Question booklet 1

- Questions 1 to 6 (52 marks)
- Answer **all** questions
- Write your answers in this question booklet
- You may write on page 10 if you need more space
- Allow approximately 65 minutes
- Approved calculators may be used — complete the box below

Examination information

Materials

- Question booklet 1
- Question booklet 2
- Formula sheet
- SACE registration number label

Instructions

- Show appropriate working and steps of logic in the question booklets
- State all answers correct to three significant figures, unless otherwise instructed
- Use black or blue pen
- You may use a sharp dark pencil for diagrams and graphical representations

Total time: 130 minutes

Total marks: 100

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Graphics calculator

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Question 1 (7 marks)

Find $\frac{dy}{dx}$ for each of the following functions. There is no need to simplify your answers.

(a) $y = \ln(5x^2 - 2x)$

(2 marks)

(b) $y = \frac{4}{x} + \sqrt{7x+1}$

(2 marks)

(c) $y = \frac{\sin(9x-3)}{8e^x}$

(3 marks)

Question 3 (7 marks)

Consider the function $f(x) = kx^2 + 1$, where k is a non-zero constant.

(a) Using first principles, show that $f'(x) = 2kx$.



(3 marks)

Figure 2 shows the graph of $y = f(x)$ for a certain value of k . The tangent to the graph at $x = 2$ is also shown. This tangent has a y -intercept at A .

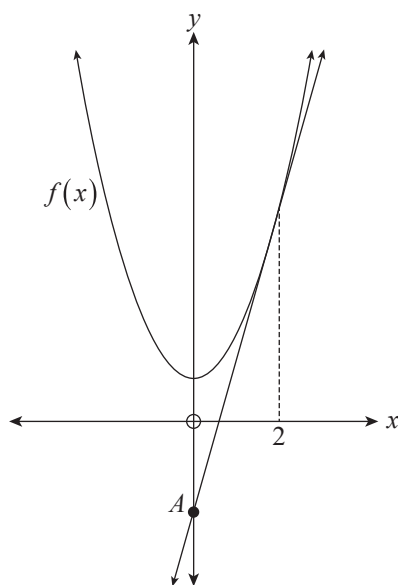
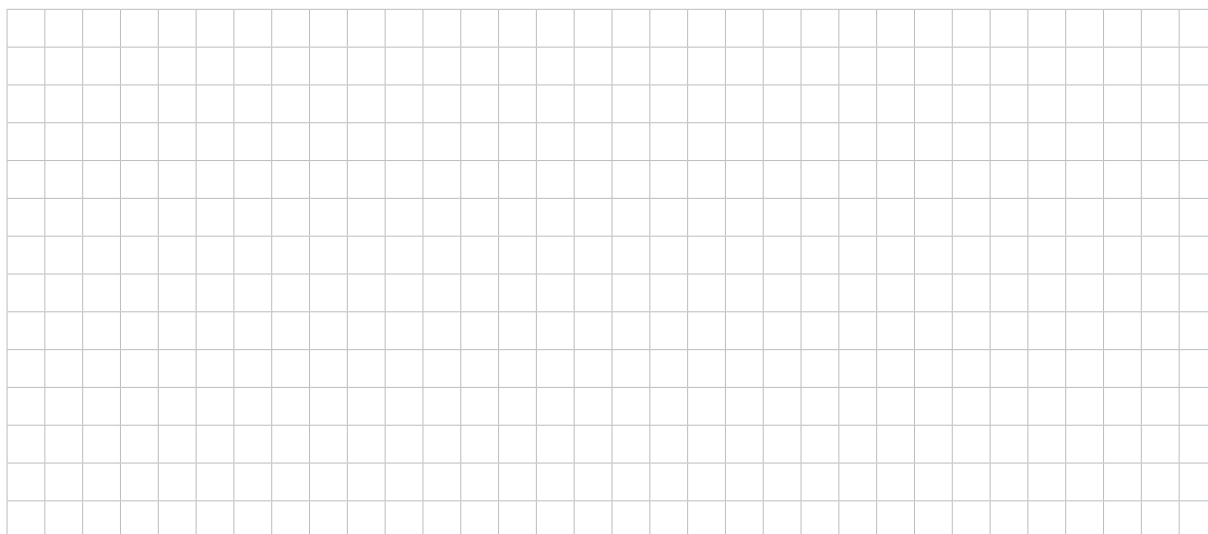


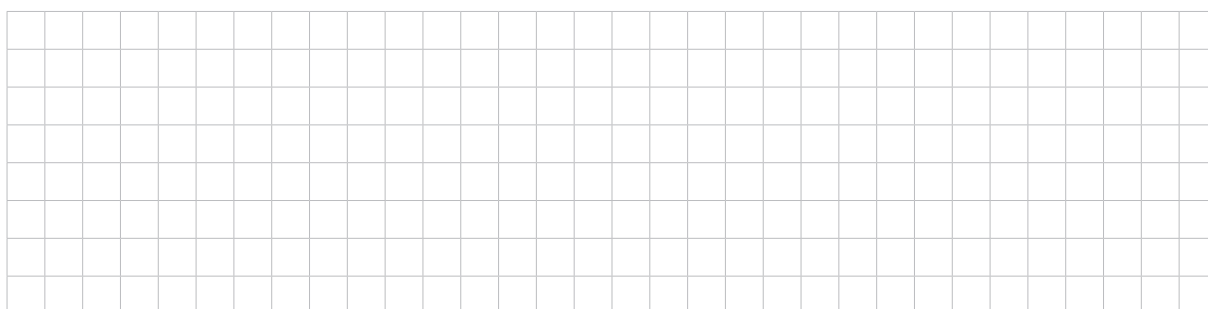
Figure 2

(b) Show that the tangent to the graph of $y = f(x)$ at $x = 2$ has the equation $y = 4kx - 4k + 1$.



(3 marks)

(c) If the y -coordinate of A is -11 , determine the value of k .



(1 mark)

You may write on this page if you need more space to finish your answers to any of the questions in this question booklet. Make sure to label each answer carefully (e.g. 4(a)(i)(3) continued).



(iii) Prove or disprove your conjecture.



(4 marks)

Question 6 begins on page 14.

Question 6 (8 marks)

Consider the graph of the function $y = f(x)$ shown in Figure 4 where $f(x)$ is defined for all real values of x . This function has exactly:

- two stationary points, located at $B(b, f(b))$ and $D(d, f(d))$
- two inflection points, located at $A(a, f(a))$ and $C(c, f(c))$
- two x -intercepts, located at $B(b, f(b))$ and $E(e, f(e))$.

As x increases for $x > d$, the slope of $f(x)$ approaches k , where $k < 0$. Additionally, $f''(x) = 0$ only at $x = a$ and $x = c$.

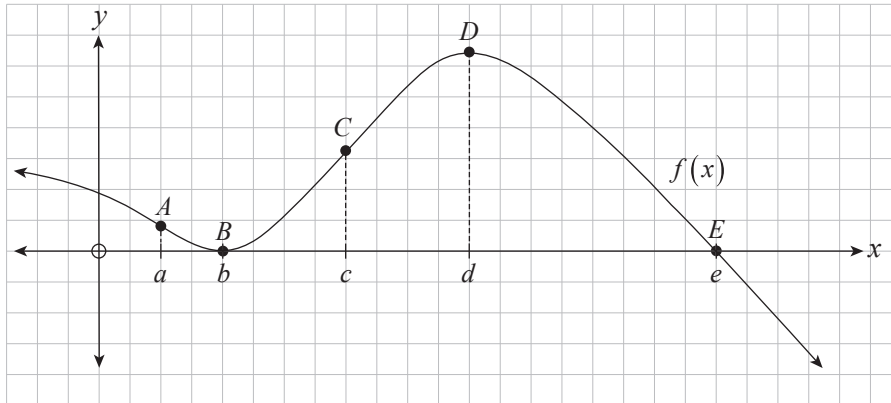


Figure 4

(a) Complete the sign diagram below for $f'(x)$.



(2 marks)

(b) (i) Which *one* statement is true? Tick the appropriate box to indicate your answer.

$f'(0) < f'(a)$

$f'(0) = f'(a)$

$f'(0) > f'(a)$

(1 mark)

(ii) Which *one* statement is true? Tick the appropriate box to indicate your answer.

$f(d) < 0$ and $f''(d) < 0$

$f(d) < 0$ and $f''(d) > 0$

$f(d) > 0$ and $f''(d) < 0$

$f(d) > 0$ and $f''(d) > 0$

(1 mark)





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Mathematical Methods

2023

Question booklet 2

- Questions 7 to 10 (48 marks)
- Answer **all** questions
- Write your answers in this question booklet
- You may write on pages 9 and 12 if you need more space
- Allow approximately 65 minutes
- Approved calculators may be used — complete the box below

2

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Copy the information from your SACE label here

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Question 8 (10 marks)

A self-balancing robot uses a device called a PID controller to stop itself from falling over after being placed on its wheels and released. Let $L(t)$ represent the angle of lean of the robot, at t seconds after release, where $L(t)$ is measured in **radians**. Side-on diagrams of this robot in Figure 6 show various angles of lean, where:

- negative values of $L(t)$ mean the robot has an angle of lean to the left
- $L(t) = 0$ means the robot is upright
- positive values of $L(t)$ mean the robot has an angle of lean to the right.

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Source: www.superroidrobots.com

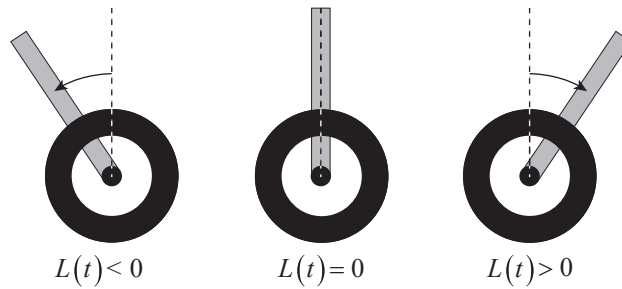


Figure 6

The PID controller's settings can be adjusted to change how long it takes the robot to reach an approximately stable vertical position after initially being placed on its wheels and released.

Engineers conducted an experiment using different settings on the PID controller, starting with Setting A.

- (a) Using Setting A, the robot was initially placed on its wheels and released. The resulting angle of lean was recorded for 6 seconds.

Using this setting, the angle of lean of the robot, $L(t)$, at t seconds after release, can be modelled by the function

$$L(t) = 0.75e^{-0.75t} \cos(4t), \text{ where } 0 \leq t \leq 6.$$

A graph of $y = L(t)$ for Setting A is shown in Figure 7.

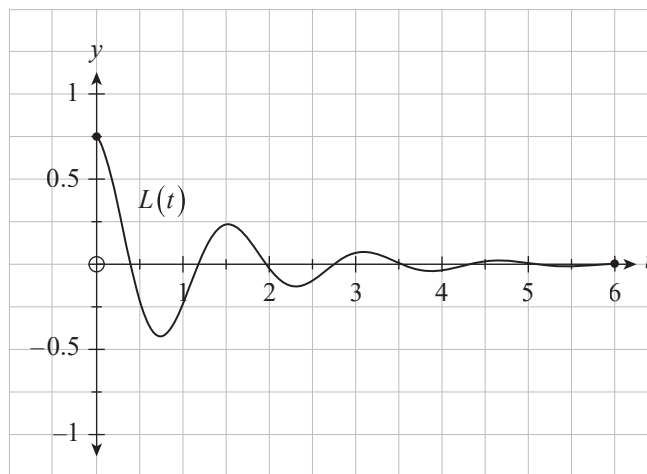
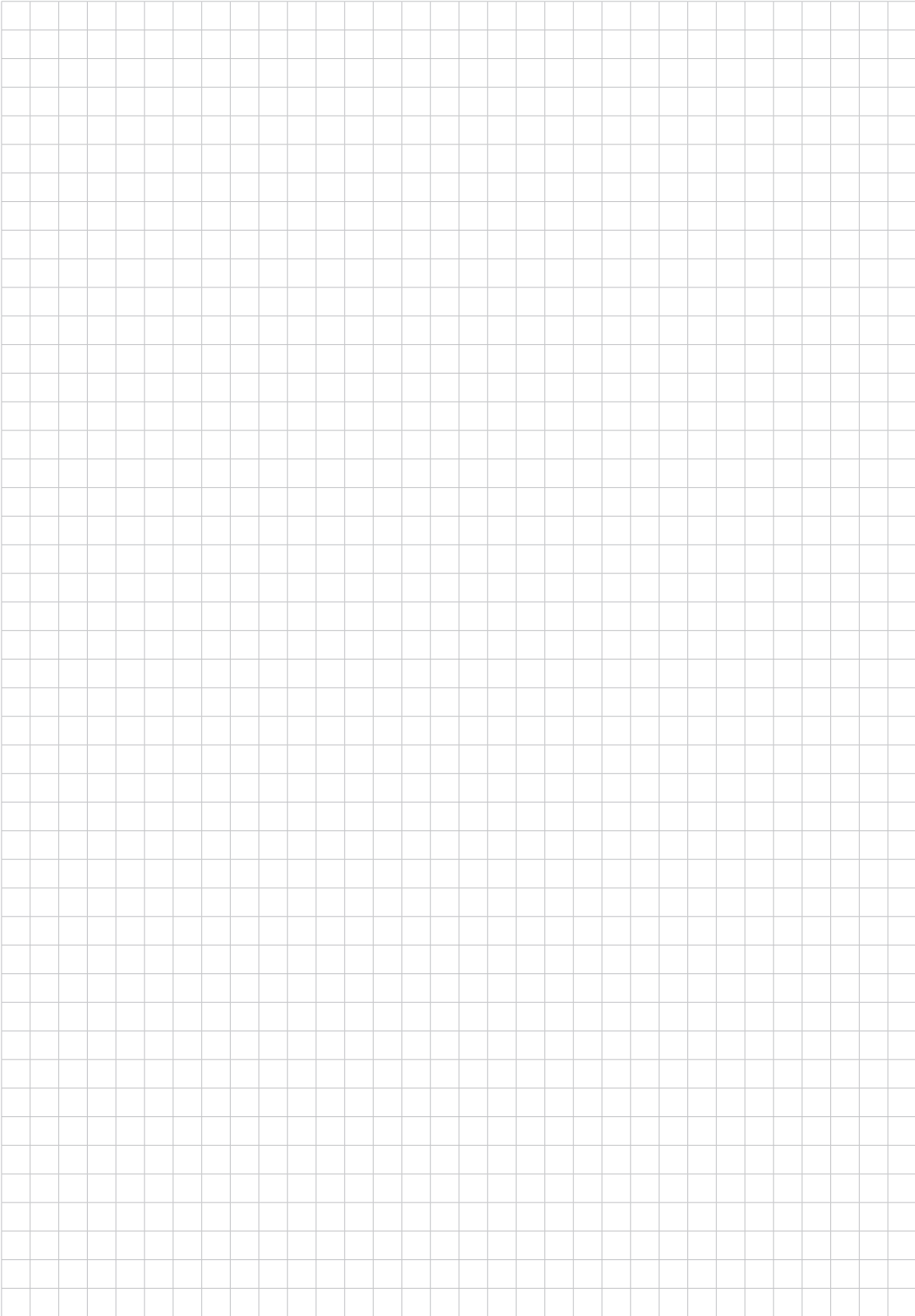


Figure 7

You may write on this page if you need more space to finish your answers to any of the questions in this question booklet. Make sure to label each answer carefully (e.g. 8(b)(ii) continued).



Question 9 (10 marks)

Consider a discrete random variable, X , which has the probability mass function

$$\Pr(X = x) = \frac{e^{-2} 2^x}{x!},$$

where x is a non-negative integer, and $x! = x \times (x-1) \times (x-2) \times \dots \times 3 \times 2 \times 1$ with $0! = 1$.

- (a) Based on the information provided, complete the table below by calculating the missing probabilities. State your answers correct to four decimal places.

$\Pr(X=0)$	$\Pr(X=1)$	$\Pr(X=2)$	$\Pr(X=3)$	$\Pr(X=4)$	$\Pr(X=5)$	$\Pr(X \geq 6)$
	0.2707	0.2707	0.1804	0.0902		0.0166

(2 marks)

A moment-generating function can be used to find key properties of a distribution without using the probability mass function directly.

For the probability mass function given above, the moment-generating function is

$$M(t) = e^{2e^t - 2}.$$

- (b) The mean, μ , of the probability mass function given above can be found using the formula

$$\mu = M'(0).$$

Show that $\mu = 2$.

(2 marks)

You may write on this page if you need more space to finish your answers to any of the questions in this question booklet. Make sure to label each answer carefully (e.g. 10(d) continued).





MATHEMATICAL METHODS FORMULA SHEET

Properties of derivatives

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

Quadratic equations

If $ax^2 + bx + c = 0$ then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Discrete random variables

The mean or expected value of a discrete random variable is:

$$\mu_X = \sum xp(x),$$

where $p(x)$ is the probability function for achieving result x .

The standard deviation of a discrete random variable is:

$$\sigma_X = \sqrt{\sum [x - \mu_X]^2 p(x)},$$

where μ_X is the expected value and $p(x)$ is the probability function for achieving result x .

Bernoulli distribution

The mean of the Bernoulli distribution is p , and the standard deviation is:

$$\sqrt{p(1-p)}.$$

Binomial distribution

The mean of the binomial distribution is np , and the standard deviation is:

$$\sqrt{np(1-p)},$$

where p is the probability of success in a single Bernoulli trial and n is the number of trials.

The probability of k successes from n trials is:

$$\Pr(X = k) = C_k^n p^k (1-p)^{n-k},$$

where p is the probability of success in the single Bernoulli trial.

Population proportions

The sample proportion is $\hat{p} = \frac{X}{n}$,

where a sample of size n is chosen, and X is the number of elements with a given characteristic.

The distribution of a sample proportion has a mean of p and a standard deviation of

$$\sqrt{\frac{p(1-p)}{n}}.$$

The upper and lower limits of a confidence interval for the population proportion are:

$$\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},$$

where the value of z is determined by the confidence level required.

Continuous random variables

The mean or expected value of a continuous random variable is:

$$\mu_X = \int_{-\infty}^{\infty} xf(x)dx,$$

where $f(x)$ is the probability density function.

The standard deviation of a continuous random variable is:

$$\sigma_X = \sqrt{\int_{-\infty}^{\infty} [x - \mu_X]^2 f(x)dx},$$

where $f(x)$ is the probability density function.

Normal distributions

The probability density function for the normal distribution with mean μ and standard deviation σ is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}.$$

All normal distributions can be transformed to the standard normal distribution with $\mu = 0$ and $\sigma = 1$ by:

$$Z = \frac{X - \mu}{\sigma}.$$

Sampling and confidence intervals

If \bar{x} is the sample mean of a sufficiently large sample, and σ is the population standard deviation, then the upper and lower limits of the confidence interval for the population mean are:

$$\bar{x} - z\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z\frac{\sigma}{\sqrt{n}},$$

where the value of z is determined by the confidence level required.