**STAGE 2 SPECIALIST MATHEMATICS**

**PROGRAM 1**

This program is for a cohort of students studying Stage 2 Specialist Mathematics. Specialist Mathematics is designed to be studied together with Stage 2 Mathematical Methods.

Technology is incorporated into aspects of all topics as appropriate.

**Topic 1 – Induction (1 Week)**

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| **Term**  **week** | **Subtopic** | **Concepts and Content** | **Assessment Task** |
| 1-1 | 1.1  Proof by Mathematical Induction | Understanding inductive proof with the initial statement and inductive step. (Revision of concept from Stage 1 Mathematics). | **SAT 1 – Induction (1.1)**  No calculator |

**Topic 2 – Complex Numbers (8 weeks)**

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| **Term**  **Week** | **Subtopic** | **Concepts and Content** | **Assessment Task** |
| 1-2 | 2.1  Cartesian and Polar Forms | Review of complex numbers   * Cartesian form. * Real and Imaginary parts.   Arithmetic of complex numbers  Describe sets of points in the complex plane   * Circular regions * Rays from the origin   Convert from Cartesian form to polar form   * Algebraic approach * Calculator approach   Properties of complex numbers        Multiplication by   * Dilation by *r* * Rotation by θ |  |
| 1-3 | Multiplication by continued.  Examples involving Stage 1 Geometry: eg. Look at rhombus properties to find  from .  Use the Principle of Mathematical Induction to prove    * .   Prove and use De Moivre’s Theorem  Consider previous proofs where all θ are equal and special case of  all equal. |  |
| 1-4 |  | De Moivre’s theorem problems  Negative powers and fractional powers.  Problem solving using polar form and De Moivre’s theorem. |  |

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| 1-5 | 2.2  The Complex (Argand) Plane | Addition of complex numbers – vector addition on complex plane  Multiplication of complex numbers (polar form)      Multiplying by  and noting the rotation.  Distance between points in the complex plane   * Geometrically * Triangle Inequality for the sum of the lengths of complex numbers e.g. * Consider also the situation of more than three sides. The possibility for another PMI proof.   Geometrical interpretation of equations and inequalities.   * Circles, lines, rays, regions |  |
| 1-6 | Cartesian equations formed for some cases.  Polar graphs   * Geometry software * Graphics calculators |  |
| 1-7 | 2.3  Roots of Complex Numbers | Solving  with c complex.  Consider   * nth roots of unity on the Argand Plane   Sum of roots by vectors – construction of n-gon. |
| 1-8 | 2.4  Factorisation of Polynomials | Review of multiplying polynomials.  Long division or synthetic division.  Equating coefficients when one factor given to lead to factorisation of polynomial.  Roots, zeros, factors.  Prove and apply Factor and Remainder Theorems.  Verifying zeros.  Factorising real cubics and quartics using complex roots and their conjugates. |  |
| 1-9 | Zeros and shape of curves.  Special examples using De Moivre’s Theorem.  Factorisation of  Revision | **SAT 2 – Complex Numbers (2.1-2.4)**  Part 1: No calculator  Part 2: Calculator permitted |

**Topic 3 – Functions and Sketching Graphs (3 weeks)**

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| Term  week | **Subtopic** | **Concepts and Content** | **Assessment Task** |
| 1-10 | 3.1  Composition of Functions | Consider composite functions and the requirements on domain and range relationships.  Finding compositions.  Check appropriate domains |  |
| 1-11 | 3-2  One-to-one Functions | Determine if a function is one-to-one.   * only when   Horizontal line test.  Inverse  of a one-to-one function.   * Unique value of domain corresponding to each element of range.   Determine the inverse of a one-to-one function.  Relationship between a function and the graph of its inverse.  Investigate symmetry about   * Software or graphics calculator approach.   Note relationship between exponential and log functions (see also Mathematical Methods). |  |
| 2-1 | 3.3  Sketching Graphs | Absolute value function notation and properties  Composite functions with absolute value  and  Reciprocal functions    where is linear, quadratic or trigonometric.  Graphs of rational functions   * Numerator and denominator both up to degree 2 with real zeros. * Asymptotic behavior via graphics calculator or other technology. | **SAT 3 – Functions and Graphs (2.1-2.4)**  Part 1: No calculator  Part 2: Calculator permitted |

**Topic 5 – Integration Techniques and Applications (6 weeks)**

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| **Term**  **week** | **Subtopic** | **Concepts and Content** | **Assessment Task** |
| 2-2 | 5.1  Integration Techniques | Integration of trigonometric and composite functions   * Use of trigonometric identities * Substitution method * Establish and use , for |  |
| 2-3 | Find and use the inverse trigonometric functions   * Restricted domain to obtain one-to-one functions.   Find and use the derivatives of inverse trigonometric functions.   * Integrate expressions of the form |  |
| 2-4 | Use partial fractions for integrating simple rational functions.  Use integration by parts. |  |
| 2-5 | 5.2  Applications of Integral Calculus | Areas between curves  Volumes of revolution   * About the x axis * About the y axis |  |
| 2-6 | Volumes of revolution (continued)   * Graphical approach for derivation of formulae   Revision | **SAT 4 – Integration (5.1-5.2)**  Calculator permitted |
| 2-7 | 5-1 and 5.2 | **INVESTIGATION** | **Investigation**  Wine Glasses |

**Topic 4 – Vectors in Three Dimensions (6 weeks)**

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| **Term**  **week** | **Subtopic** | **Concepts and Content** | **Assessment Task** |
| 2-8 | 4.1  The Algebra of Vectors in Three Dimensions | Review of vectors from Stage 1 Topic 3.   * Developing to 3D * Unit vectors   Continue algebra of vectors in 3D. |  |
| 2-9 | 4.2  Vector and Cartesian Equations | Cartesian coordinates   * plotting points * equations of spheres   Equation of a line in 2D and 3D   * vector equation and parametric form * Cartesian form * Parallel, perpendicular and skew lines * Closest point on line to another point * Distance between skew lines * Angle between two lines   Path of two particles   * Using vectors as functions of time, determine whether particle paths cross or meet. |  |
| 2-10 | Scalar (dot) product and vector (cross) product   * Use coordinates of length and angle   Context: Perpendicular and parallel vectors  Vector (cross) product   * calculation using the determinant (2x2 and 3x3) * geometric relevance   is the area of a parallelogram, sides **a** and **b**.  Equation of a plane   * Develop using vector equations * Intersection of a line and a plane.   Lines parallel to or coincident with planes. |  |
| 3-1 | Find the point on a given plane closest to a point in space.  Equality of vectors   * Seen using opposite sides of parallelogram   Coordinate systems and position vectors  Triangle Inequality  Connection from Sub-topic 2.2  Vector Proof   * Establishing parallelism, perpendicularity, properties of intersections   If where  are not parallel, then |  |
| 3-2 | 4.3  Systems of Linear Equations | General form of system of equations  Elementary techniques of elimination to solve up to 3x3 system  Possible solutions and geometric interpretation (continued).   * Algebraic and geometric descriptions of * unique solution * no solution * infinitely many solutions |  |
| 3-3 | Finding intersection of two or more planes | **SAT 5 – Vectors (4.1-4.3)**  Calculator permitted |

**Topic 6 – Rates of Change and Differential Equations (7 weeks)**

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| **Term**  **week** | **Subtopic** | **Concepts and Content** | **Assessment Task** |
| 3-4 | 6.1  Implicit Differentiation | Implicit differentiation (following on from Mathematical Methods)  Finding gradients of curves in implicit form.  Derivation of the derivative of the natural log function. |  |
| 3-5 | 6.2  Differential Equations | Related Rates   * Examples of calculating   Differential Equations   * Solving * Solving |  |
| 3-6 | Slope fields   * For first order DEs * Graph from slope field manually and using software or calculators |  |
| 3-7 | Modelling with DEs   * Separable DEs examples * Logistic |  |
| 3-8 | 6.3  Pairs of Varying Quantities – Polynomials of Degree 1 to 3 | Curves produced by moving point  Coordinate representation (parametric)  Quantities of the form  Vector representation with t as time      Examples   * Objects in free flight * Bézier curves |  |
| 3-9 | 6.4  Related Rates, Velocity and Tangents | For a moving point   * **V** = instantaneous velocity * Cartesian equation from parametric * Velocity vector is tangent to the curve * Speed of moving point      * Arc length of path traced out |  |
| 3-10 | 6.5  Trigonometric Parametrisations | Point moving with unit speed around the unit circle has position  Consider   * Use Arc length formula to establish circumference of a circle | **SAT 6 – Rates of Change and Differential Equations (6.1-6.5)**  Calculator permitted |

**Revision**

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| Term  week | **Subtopic** | **Concepts and Content** | **Assessment Task** |
| Term 4 |  | Revision / Swot Vac / Exam |  |

**SUGGESTED ALLOCATION OF TIME**Topic 1: Mathematical Induction (1 week)  
Topic 2: Complex Numbers (8 weeks)  
Topic 3: Functions and Sketching Graphs (3 weeks)   
Topic 5: Integration Techniques and Applications (6 weeks)  
Topic 4: Vectors in Three Dimensions (6 weeks)  
Topic 6: Rates of Change and Differential Equations (7 weeks)

**ASSESSMENT**

School-based Assessment (70%)

* Assessment Type 1: Skills and Applications Tasks (50%)
* Assessment Type 2: Mathematical Investigation (20%)

External Assessment (30%)

* Assessment Type 3: Examination (30%).