PART ONE – NO CALCULATOR OR NOTES 25 marks **35 minutes**

$ax^2 + bx + c = 0$

$\therefore x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

QUESTION 1 **(6 marks)**

Solve algebraically for x:

(a) $21x^2 + 14x = 0$

(2 marks)

(b) $4x - \frac{6}{x} = -5$

(4 marks)

QUESTION 2 **(6 marks)**

Give *exact* solutions to each of the following equations by using the method stated, expressing your answer in simplest surd form where appropriate.

(a) Solve $x^2 - 8x + 4 = 0$ by completing the square.

(3 marks)

(b) Expand and simplify and then use the quadratic formula to solve $(x + 3)(2x + 1) = 9$.

(3 marks)

QUESTION 3 (6 marks)

(a) Using the discriminant only, state the nature of the solutions for $-2x^2 + 5x - 2 = 0$.

(2 marks)

(b) Using the discriminant only, state the relationship between the graph of $y = x^2 + 3x + 4$ and the X axis.

(2 marks)

(c) Find the sum and product of the roots of the equation $3x^2 - 6x + 2 = 0$ without solving the equation.

(2 marks)

QUESTION 4 **(7 marks)**

Consider $3x^2 - 2x + c = 0$

(a) Show that the discriminant is $\Delta = 4 - 12c$.

(1 mark)

(b) *Hence* find values of c for which the equation has:

(i) a repeated root ;

(2 marks)

(ii) no real roots ;

(2 marks)

(iii) has rational roots. (Only 1 value needs to be given)

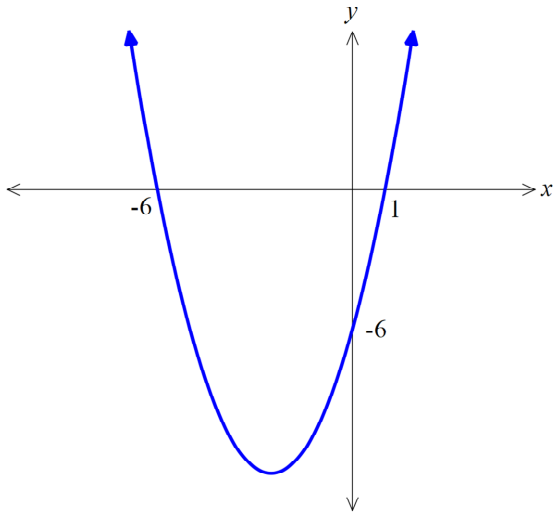
(2 marks)

PART TWO – CALCULATOR AND NOTES ALLOWED **34 marks**

QUESTION 5 **(6 marks)**

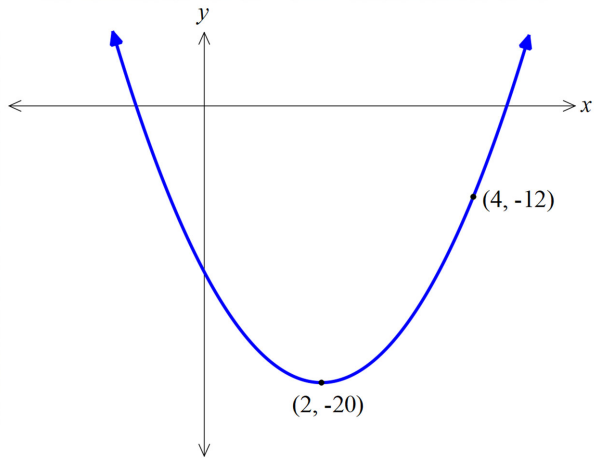
Find the equation of each quadratic, given the following information.

(a)



(3 marks)

(b)



(3 marks)

QUESTION 6 **(8 marks)**

Consider the quadratic $y = 4x^2 - 8x - 5$;

(a) find the axis of symmetry and the vertex.

(2 marks)

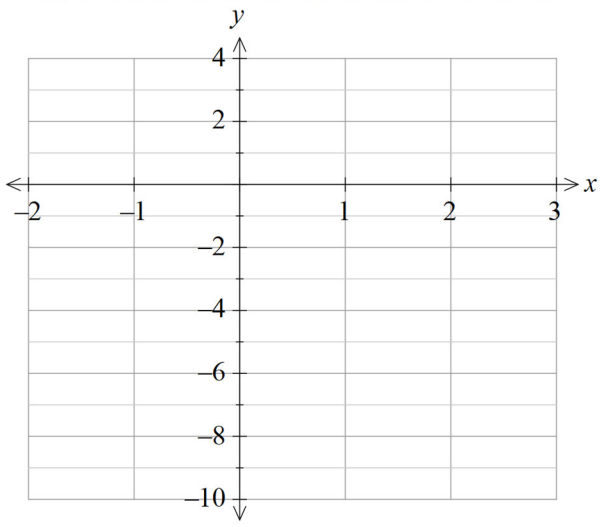
(b) (i) State the *y* intercept.

(1 mark)

(ii) Find the x intercepts.

(2 marks)

(c) Showing clearly the information found in parts (a) and (b) sketch the graph of the quadratic $y = 4x^2 - 8x - 5$ on the axes given below.



(3 marks)

QUESTION 7 **(9 marks)**

(a) Find algebraically the co-ordinates of the point(s) of intersection of:

$y = 2 - 2x$ and $y = 3x^2 - 5x - 4$

(4 marks)

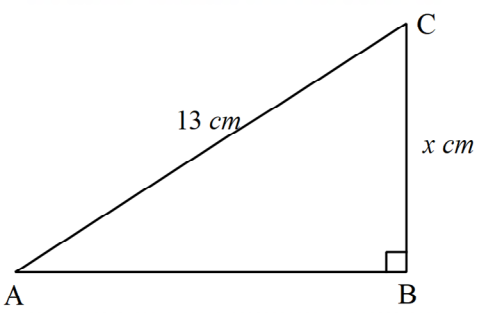
(b) Consider the curve $y = -x^2 + 3x - 6$ and the line $y = mx - 2$.

Find the values of m for which the line is tangent to the curve.

(5 marks)

QUESTION 8 **(6 marks)**

A right angled triangle is shown with hypotenuse of length 13 cm and the side BC is marked $x\ cm$.



(a) Given that side AB is 7 cm longer than BC, write AB in terms of x.

(1 mark)

(a) Explain why $2x^2 + 14x - 120 = 0$.

(2 marks)

(b) (i) Using part (b) find the value of x.

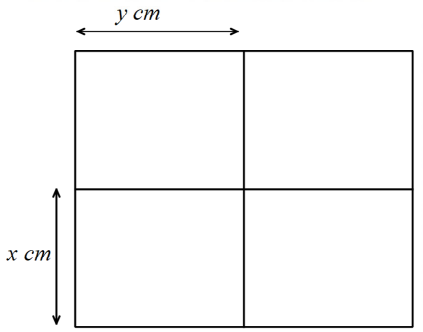
(2 marks)

(ii) Find the area of the triangle.

(1 mark)

QUESTION 9 **(5 marks)**

Fencing of 840 metres is used to construct 4 rectangular paddocks of equal size as shown in the diagram.



(a) Explain why $x + y = 140$.

(1 mark)

(b) Show that the area of each pen is given by $A = 140x - x^2$.

(1 mark)

(c) (i) If the area enclosed by each pen is to be maximized, what are the dimensions of each pen?

(2 marks)

(ii) Describe the shape of each pen.

(1 mark)

PART ONE – NO CALCULATOR OR NOTES 25 marks **35 minutes**

$ax^2 + bx + c = 0$

$\therefore x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

QUESTION 1 **(6 marks)**

Solve algebraically for x:

(a) $21x^2 + 14x = 0$

$7x(3x + 2) = 0$ ✓

$\therefore x = 0$ or $-\frac{2}{3}$ ✓

(2 marks)

(b) $4x - \frac{6}{x} = -5$

$4x^2 - 6 + 5x = 0$

$\therefore 4x^2 + 5x - 6 = 0$ ✓

$\therefore (4x - 3)(x + 2) = 0$ ✓ ✓

$\therefore x = \frac{3}{4}$ or $x = -2$ ✓

(4 marks)

QUESTION 2 **(6 marks)**

Give *exact* solutions to each of the following equations by using the method stated, expressing your answer in simplest surd form where appropriate.

(a) Solve $x^2 - 8x + 4 = 0$ by completing the square.

$(x - 4)^2 = 12$ ✓

$\therefore x - 4 = \pm\sqrt{12} = \pm 2\sqrt{3}$ ✓

$\therefore x = 4 \pm 2\sqrt{3}$ ✓

(3 marks)

(b) Expand and simplify and then use the quadratic formula to solve $(x+3)(2x+1)=9$.

$2x^2+7x+3=9$

$\therefore 2x^2+7x-6=0$ ✓

$\therefore x=-\frac{7}{4}$ ✓ $\pm\frac{\sqrt{49+48}}{4}$ ✓

$\therefore x=-\frac{7}{4}\pm\frac{\sqrt{97}}{4}$

(3 marks)

QUESTION 3 (6 marks)

(a) Using the discriminant only, state the nature of the solutions for $-2x^2+5x-2=0$.

$\Delta=25-4(-2)(-2)=9>0$ ✓

$\therefore$ 2 rational distinct solutions ✓

(2 marks)

(b) Using the discriminant only, state the relationship between the graph of $y=x^2+3x+4$ and the X axis.

$\Delta=9-16<0$ ✓

$\therefore$ Entirely above X axis $(ie\ +ve$ definite) ✓

(2 marks)

(c) Find the sum and product of the roots of the equation $3x^2-6x+2=0$ without solving the equation.

Sum roots $=-\frac{-6}{3}=2$ ✓

Product roots $=\frac{2}{3}$ ✓

(2 marks)

QUESTION 4 **(7 marks)**

Consider $3x^2 - 2x + c = 0$

(a) Show that the discriminant is $\Delta = 4 - 12c$.

$\Delta = (-2)^2 - 4 \times 3 \times c$ ✓ $= 4 - 12c$

(1 mark)

(b) *Hence* find values of c for which the equation has:

(i) a repeated root ;

$4 - 12c = 0$ ✓

$\therefore c = \frac{1}{3}$ ✓

(2 marks)

(ii) no real roots ;

$4 - 12c < 0$ ✓

$\therefore 4 < 12c$

$\therefore \frac{1}{3} < c$ ✓ or write as $c > \frac{1}{3}$

(2 marks)

(iii) has rational roots. (Only 1 value needs to be given)

$4 - 12c = 1$ or 4 or 9 etc ✓

$\therefore 12c = 3$ or 0 or -5

$\therefore c = \frac{1}{4}$ or 0 or $-\frac{5}{12}$ etc ✓ (*One* value is all that is needed)

(2 marks)

PART TWO – CALCULATOR AND NOTES ALLOWED **34 marks**

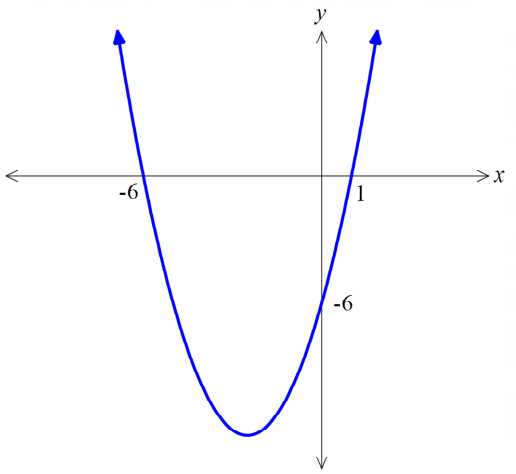
QUESTION 5 **(6 marks)**

Find the equation of each quadratic, given the following information.

(a) $y = a(x + 6)(x - 1)$ ✓ ✓

$(0, -6) \Rightarrow -6 = a(6)(-1) \Rightarrow a = 1$ ✓

$\therefore y = (x + 6)(x - 1)$

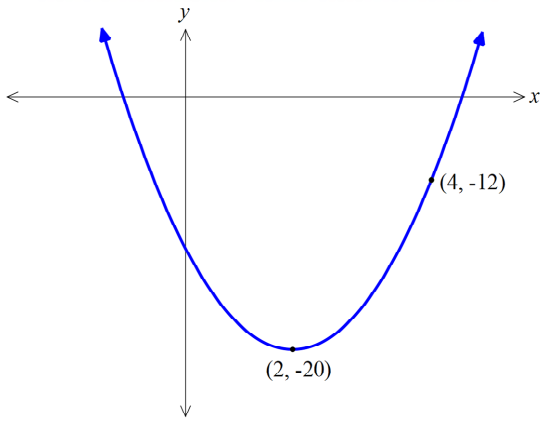


(3 marks)

(b) $y = a(x - 2)^2 - 20$ ✓ ✓

$(5, 0) \Rightarrow 0 = a(9) - 20 \Rightarrow a = \frac{20}{9}$ ✓

$\therefore y = \frac{20}{9}(x - 2)^2 - 20$



(3 marks)

QUESTION 6 **(8 marks)**

Consider the quadratic $y = 4x^2 - 8x - 5$;

(a) find the axis of symmetry and the vertex.

$x = -\frac{-8}{8} = 1$ Axis of symmetry ✓

$y = 4 - 8 - 5 = -9$ or gcalc ⇒ Vertex $(1, -9)$ ✓

(2 marks)

(b) (i) State the *y* intercept.

$(0\ ,-5)$ ✓

(1 mark)

(ii) Find the x intercepts.

$4x^2 - 8x - 5 = 0$

$\therefore (2x - 5)(2x + 1) = 0$

$\therefore x = \frac{5}{2}$ or $-\frac{1}{2}$ ✓ ✓ or use gcalc

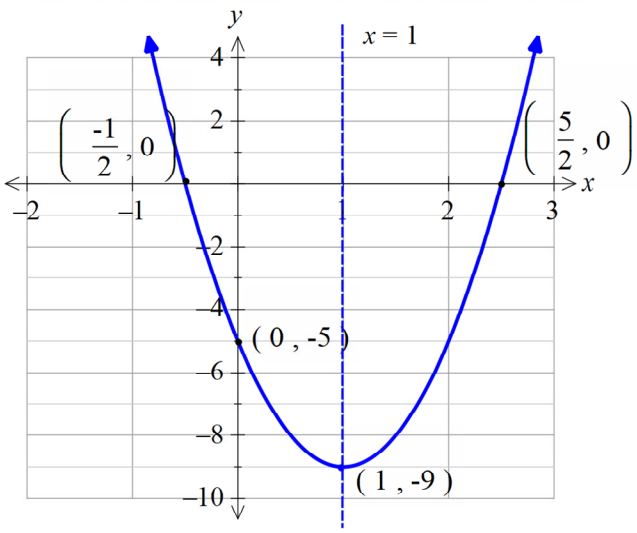
(2 marks)

(c) Showing clearly the information found in parts (a) and (b) sketch the graph of the quadratic $y = 4x^2 - 8x - 5$ on the axes given below.

Y Intercept and *X* intercepts ✓

Vertex ✓

Shape ✓



(3 marks)

QUESTION 7 **(9 marks)**

(a) Find algebraically the co-ordinates of the point(s) of intersection of:

$y = 2 - 2x$ and $y = 3x^2 - 5x - 4$

$2 - 2x = 3x^2 - 5x - 4$ ✓

$\therefore 0 = 3x^2 - 3x - 6$ ✓

$\therefore 0 = 3(x^2 - x - 2) = 3(x - 2)(x + 1)$

$\therefore x = 2$ or -1 ✓ or use gcalc

$\therefore (2, -2)$ or $(-1, 4)$ ✓

(4 marks)

(b) Consider the curve $y = -x^2 + 3x - 6$ and the line $y = mx - 2$.

Find the values of m for which the line is tangent to the curve.

$-x^2 + 3x - 6 = mx - 2$ ✓

$\therefore 0 = x^2 + (m - 3)x + 4$ ✓

$\Delta = (m - 3)^2 - 16 = 0$ (Tangent) ✓

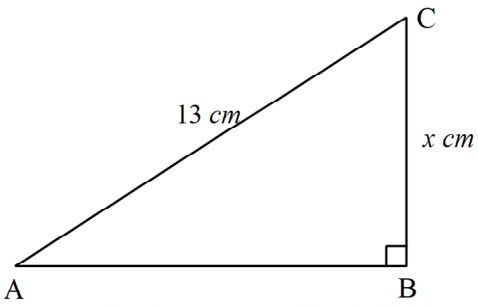
$\therefore m - 3 = \pm 4$

$\therefore m = 7$ or -1 ✓ ✓

(5 marks)

QUESTION 8 **(6 marks)**

A right angled triangle is shown with hypotenuse of length 13 cm and the side BC is marked $x\ cm$.



(a) Given that side AB is 7 cm longer than BC, write AB in terms of x.

$AB = x + 7$ ✓

(1 mark)

(b) Explain why $2x^2 + 14x - 120 = 0$.

$x^2 + (x+7)^2 = = 13^2$ (Pythagoras) ✓

$\therefore 2x^2 + 14x + 49 = 169$ ✓

$\Rightarrow 2x^2 + 14x - 120 = 0$

(2 marks)

(c) (i) Using part (b) find the value of x.

$2(x^2 + 7x - 60) = 0$

$\therefore 2(x+12)(x-5) = 0$

$\therefore x = -12$ or $x = 5$ ✓ But $x > 0$

$\Rightarrow x = -12$ not feasible ✓ $\Rightarrow$ Only value is $x = 5$ or use gcalc

(2 marks)

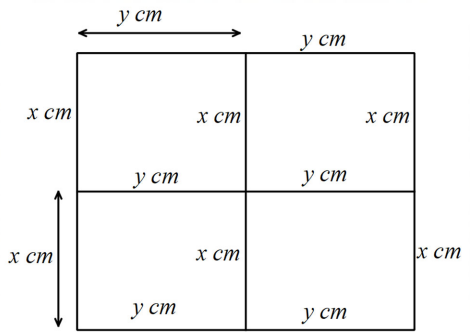
(ii) Find the area of the triangle.

$\text{Area } \Delta = \frac{5 \times 12}{2} = 30\ cm^2$ ✓

(1 mark)

QUESTION 9 **(5 marks)**

Fencing of 840 metres is used to construct 4 rectangular paddocks of equal size as shown in the diagram.



(a) Explain why $x + y = 140$.

Clearly from diagram fencing = $6x + 6y = 840$ ✓

$\therefore x + y = \frac{840}{6} = 140$

(1 mark)

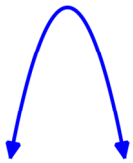
(b) Show that the area of each pen is given by $A = 140x - x^2$.

$x + y = 140 \Rightarrow y = 140 - x$

Area of each pen = $xy = x(140 - x)$ ✓ $= 140x - x^2$

(1 mark)

(c) (i) If the area enclosed by each pen is to be maximized, what are the dimensions of each pen?



Maximum when $x = -\frac{-140}{2 \times 1} = 70$ ✓

$\therefore y = 140 - x = 140 - 70 = 70$ ✓

$\therefore$ Dimensions are $70m$ by $70\ m$

(2 marks)

(ii) Describe the shape of each pen.

Since $x = y = 70 \Rightarrow$ Square ✓

(1 mark)