DRAFT FOR ONLINE CONSULTATION

Mathematical Methods

Subject Outline

Stage 1 and Stage 2

Version 1.0 – Draft for Online Consultation
Ref: A375006
11 March 2015 – 17 April 2015

Government of South Australia

SACE Board of SA

CONTENTS

INTRODUCTION

SUBJECT DESCRIPTION

Mathematical Methods is a 10-credit subject or a 20-credit subject at Stage 1, and a 20-credit subject at Stage 2.

Mathematical Methods develops an increasingly complex and sophisticated understanding of calculus and statistics. By using functions, their derivatives and integrals, and by mathematically modeling physical processes, students develop a deep understanding of the physical world through a sound knowledge of relationships involving rates of change. Students use statistics to describe and analyse phenomena that involve uncertainty and variation.

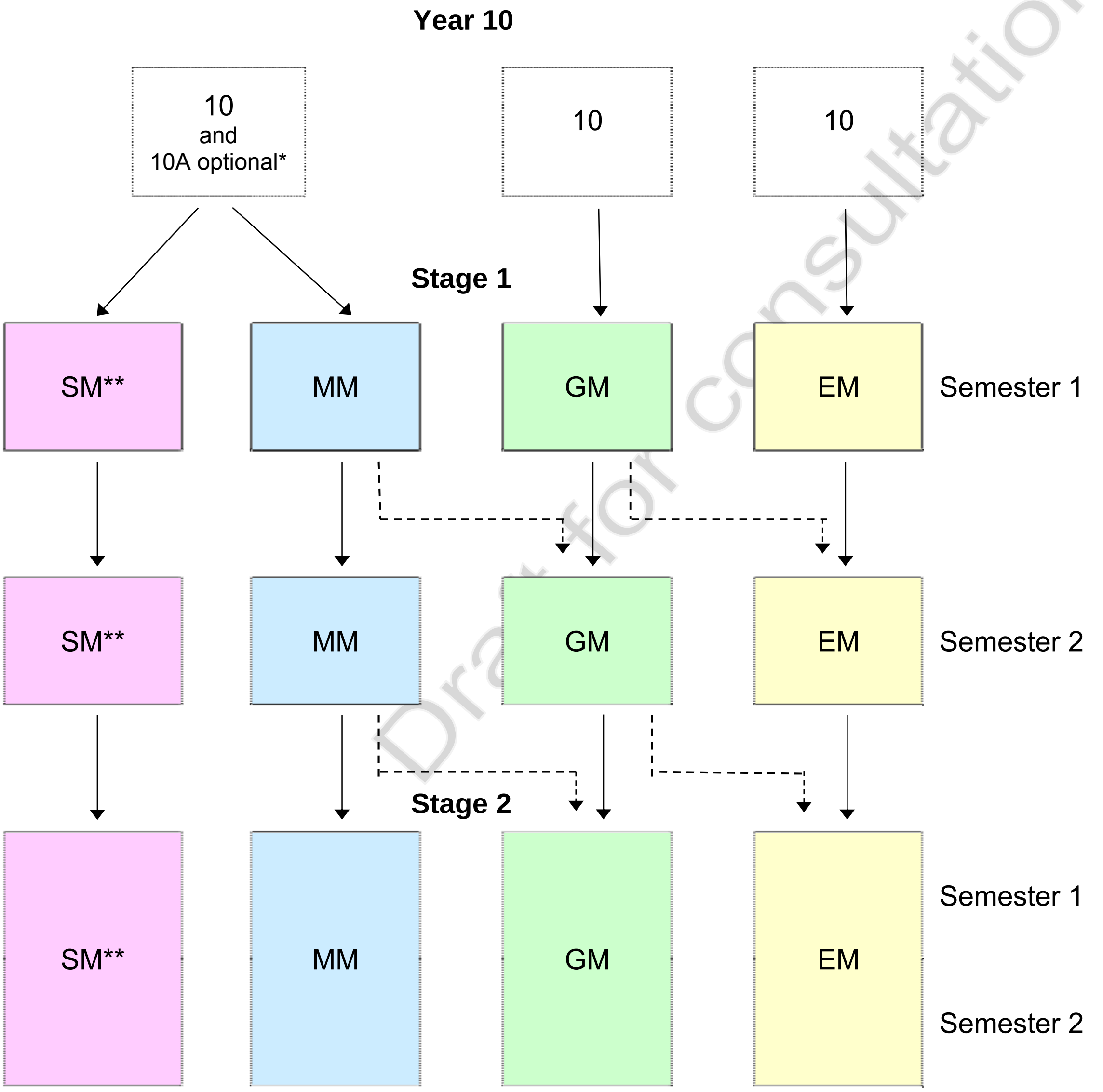
Mathematical Methods provides the foundation for further study in mathematics, economics, computer sciences, and the sciences. It prepares students for courses and careers that may involve the use of statistics, such as health or social sciences. When studied together with Specialist Mathematics, this subject can be a pathway to engineering, space science, and laser physics.

MATHEMATICAL OPTIONS

The diagram below represents the possible mathematical options that students might select as Stage 1 and Stage 2 subjects.

Solid arrows indicate the mathematical options that lead to completion of each subject at Stage 2. Dotted arrows indicate a pathway that may provide sufficient preparation for an alternative Stage 2 mathematics subject.

SM	Specialist Mathematics
MM	Mathematical Methods
GM	General Mathematics
EM	Essential Mathematics



Notes: * Although it is advantageous for students to study Australian Curriculum 10 and 10A in Year 10, the 10A curriculum *per se* is not a prerequisite for the study of Specialist Mathematics and Mathematical Methods. The essential aspects of 10A are included into the curriculum for Specialist Mathematics and Mathematical Methods.

** Mathematical Methods can be studied as a single subject; however, Specialist Mathematics is designed to be studied together with Mathematical Methods.

CAPABILITIES

The capabilities connect student learning within and across subjects in a range of contexts. They include essential knowledge and skills that enable people to act in effective and successful ways.

The SACE identifies seven capabilities. They are:
- literacy
- numeracy
- information and communication technology capability
- critical and creative thinking
- personal and social capability
- ethical understanding
- intercultural understanding.

Literacy

In this subject students develop their literacy capability by, for example:
- communicating mathematical reasoning and ideas for different purposes, using appropriate language and representations, such as symbols, equations, tables, and graphs
- interpreting and responding to appropriate mathematical language and representations
- analysing information and explaining mathematical results.

Mathematics provides a specialised language to describe and analyse phenomena. It provides a rich context for students to extend their ability to read, write, visualise, and talk about situations that involve investigating and solving problems.

Students apply and extend their literacy skills and strategies by using verbal, graphic, numerical, and symbolic forms of representing problems and displaying statistical information. Students learn to communicate their findings in different ways, using different systems of representation.

Numeracy

Being numerate is essential for participating in contemporary society. Students need to reason, calculate, and communicate to solve problems. Through the study of mathematics, they understand and use mathematical skills, concepts, and technologies in a range of contexts that can be applied to:
- using measurement in the physical world
- gathering, representing, interpreting, and analysing data
- using spatial sense and geometric reasoning
- investigating chance processes
- using number, number patterns, and relationships between numbers
- working with graphical, statistical and algebraic representations, and other mathematical models.

Information and communication technology capability

In this subject students develop their information and communication technology (ICT) capability by, for example:
- understanding the role of electronic technology in the study of mathematics
- making informed decisions about the use of electronic technology
- understanding the mathematics involved in computations carried out using technologies, so that reasonable interpretations can be made of the results.

Students extend their skills in using technology effectively and processing large amounts of quantitative information.

Students use ICT to extend their theoretical mathematical understanding and apply mathematical knowledge to a range of problems. They use software relevant for study and/or workplace contexts. This may include tools for statistical analysis, algorithm generation, data representation and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, to use data, address problems, and operate systems in particular situations.

Critical and creative thinking

In this subject students develop critical and creative thinking by, for example:

- building confidence in applying knowledge and problem-solving skills in a range of mathematical contexts
- developing mathematical reasoning skills to think logically and make sense of the world
- understanding how to make and test projections from mathematical models
- interpreting results and drawing appropriate conclusions
- reflecting on the effectiveness of mathematical models, including the recognition of assumptions, strengths, and limitations
- using mathematics to solve practical problems and as a tool for learning
- making connections between concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas
- thinking abstractly, making and testing conjectures, and explaining processes.

Problem-solving in mathematics builds students' depth of conceptual understanding and supports development of critical and creative thinking. Learning through problem-solving helps students when they encounter new situations. They develop their creative and critical thinking capability by listening, discussing, conjecturing, and testing different strategies. They learn the importance of self-correction in building their conceptual understanding and mathematical skills.

Personal and social capability

In this subject students develop their personal and social capability by, for example:

- arriving at a sense of self as a capable and confident user of mathematics through expressing and presenting ideas in a variety of ways
- appreciating the usefulness of mathematical skills for life and career opportunities and achievements
- understanding the contribution of mathematics and mathematicians to society.

The elements of personal and social competence relevant to mathematics include the application of mathematical skills for informed decision-making, active citizenship, and effective self-management. Students build their personal and social competence in mathematics through setting and monitoring personal and academic goals, taking initiative, and building adaptability, communication, and teamwork.

Students use mathematics as a tool to solve problems they encounter in their personal and working lives. They acquire a repertoire of strategies and build the confidence needed to:

- meet the challenges and innovations of a rapidly changing world
- be the designers and innovators of the future, and leaders in their fields.

Ethical understanding

In this subject students develop their ethical understanding by, for example:

- gaining knowledge and understanding of ways in which mathematics can be used to support an argument or point of view
- examining critically ways in which the media present particular perspectives
- sharing their learning and valuing the skills of others
- considering the social consequences of making decisions based on mathematical results
- acknowledging and learning from errors rather than denying findings and/or evidence.

Areas of ethical understanding relevant to mathematics include issues associated with ethical decision-making and working collaboratively as part of students' mathematically related explorations. They develop ethical understanding in mathematics through considering social responsibility in ethical dilemmas that may arise when solving problems in personal, social, community, and/or workplace contexts.

Intercultural understanding

In this subject students develop their intercultural understanding by, for example:

- understanding mathematics as a body of knowledge that uses universal symbols which have their origins in many cultures
- understanding how mathematics assists individuals, groups, and societies to operate successfully across cultures in the global, knowledge-based economy.

Mathematics is a shared language that crosses borders and cultures, and is understood and used globally.

Students read about, represent, view, listen to, and discuss mathematical ideas. They become aware of the historical threads from different cultures that have led to the current bodies of mathematical knowledge. These opportunities allow students to create links between their own language and ideas and the formal language and symbols of mathematics.

ABORIGINAL AND TORRES STRAIT ISLANDER KNOWLEDGE, CULTURES, AND PERSPECTIVES

In partnership with Aboriginal and Torres Strait Islander communities, and schools and school sectors, the SACE Board of South Australia supports the development of high-quality learning and assessment design that respects the diverse knowledge, cultures, and perspectives of Indigenous Australians.

The SACE Board encourages teachers to include Aboriginal and Torres Strait Islander knowledge and perspectives in the design, delivery, and assessment of teaching and learning programs by:

- providing opportunities in SACE subjects for students to learn about Aboriginal and Torres Strait Islander histories, cultures, and contemporary experiences
- recognising and respecting the significant contribution of Aboriginal and Torres Strait Islander peoples to Australian society

- drawing students' attention to the value of Aboriginal and Torres Strait Islander knowledge and perspectives from the past and the present
- promoting the use of culturally appropriate protocols when engaging with and learning from Aboriginal and Torres Strait Islander peoples and communities.

SACE NUMERACY REQUIREMENT

Completion of 10 or 20 credits of Stage 1 Mathematical Methods with a C grade or better, or 20 credits of Stage 2 Mathematical Methods with a C- grade or better, will meet the numeracy requirement of the SACE.

Stage 1
Mathematical Methods

LEARNING SCOPE AND REQUIREMENTS

LEARNING REQUIREMENTS

The learning requirements summarise the knowledge, skills, and understanding that students are expected to develop and demonstrate through their learning in Stage 1 Mathematical Methods.

In this subject, students are expected to:

1. understand mathematical concepts, demonstrate mathematical skills, and apply mathematical techniques
2. investigate and analyse mathematical information in a variety of contexts
3. think mathematically by posing questions and solving problems, including making and testing conjectures
4. interpret results, draw conclusions, and determine the reasonableness of solutions in context
5. make discerning use of electronic technology
6. communicate mathematically and present mathematical information in a variety of ways.

CONTENT

Stage 1 Mathematical Methods may be studied as a 10-credit subject or a 20-credit subject.

Mathematical Methods at Stage 1 builds on the mathematical knowledge, understanding, and skills that students have developed in Number and Algebra, Measurement and Geometry, and Statistics and Probability during Year 10.

Stage 1 Mathematical Methods is organised into topics that broaden students' mathematical experience, and provide a variety of contexts for incorporating mathematical arguments and problem solving. The topics provide a blending of algebraic and geometric thinking. In this subject there is a progression of content, applications, and level of sophistication and abstraction.

Key concepts from 10A Mathematics in the Australian Curriculum required for the study of Mathematical Methods and Specialist Mathematics, that is, indices, quadratics, trigonometry, mean and standard deviation, graphing, and logarithms, have been incorporated in Stage 1 Mathematical Methods.

Stage 1 Mathematical Methods consists of the following list of six topics:

- Topic 1: Functions and graphs
- Topic 2: Trigonometry
- Topic 3: Counting and Probability
- Topic 4: Statistics
- Topic 5: Growth and Decay
- Topic 6: Introduction to Differential Calculus.

There are two types of topics: major and minor. Major topics require a longer time to develop the key concepts.

Topics 1, 2, 5, and 6 are major topics.
Topics 3 and 4 are minor topics.

Programming

For a 10-credit subject students study three of the topics:

- 2 major topics (Topics 1, 2, 5, or 6); and
- 1 minor topic (Topic 3 or 4).

For a 20-credit subject students study all six topics.

The topics selected can be sequenced and structured to suit individual cohorts of students. The suggested order of the topics provided in the list is a guide only. However, when Mathematical Methods studied in conjunction with Specialist Mathematics, consideration should be given to appropriate sequencing of the topics across the two subjects.

Each topic consists of a number of subtopics. These are presented in the subject outline in two columns, as a series of ‘key questions and key concepts’ side by side with ‘considerations for developing teaching and learning strategies’.

The ‘key questions and key concepts’ cover the prescribed content for teaching, learning, and assessment in this subject. The ‘considerations for developing teaching and learning strategies’ are provided as a guide only.

A problem-based approach is integral to the development of the mathematical models and associated key concepts in each topic. Through key questions students deepen their understanding of concepts and processes that relate to the mathematical models required to address the problems posed.

The ‘considerations for developing teaching and learning strategies’ present problems and guidelines for sequencing the development of the ‘key questions and key concepts’. They also give an indication of the depth of treatment and emphases required.

Students use electronic technology, where appropriate, to enable complex problems to be solved efficiently.

Topic 1: Functions and Graphs (Major Topic)

This topic provides students with the algebraic concepts and techniques required for a successful introduction to the study of calculus. Simple relationships between variable quantities are used to introduce the key concepts of a function and its graph.

The emphasis is on describing, sketching, interpreting, and discussing the behaviour of graphs that arise from everyday situations. Students focus on describing and explaining the characteristics and behaviour of a graph in relation to the situation being modelled.

The investigation of links between the algebraic and graphical representations of functions relies on the use of technology for the production of graphs of mathematical functions. Students test their conjectures using many examples, without plotting graphs themselves.

Students examine mathematical models arising from different situations that can be described algebraically, using polynomial functions. As students gain a sound understanding of the graphical behaviour of these functions, they develop their skills in the algebraic manipulation of polynomials. The links between these two concepts are strengthened by the use of electronic technology.

Sub-topic 1.1: Lines and Linear Relationships

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How is it possible to work with linear relationships, given some data or a description of a situation? • Direct proportion • Linear related variables.	The functions used are drawn from everyday contexts (e.g. taxi fares, simple interest, water rates, conversion graphs, telephone charges).
How can all the points on a straight line be described mathematically? • The equation of a straight line - from two points - from a slope and a point - parallel and perpendicular to a given line through some other point • Length of a line segment • Coordinates of the mid-point of a line segment.	In the context of a city environment, straight lines occur as roads, storm water drains, gas pipelines, and so on. The location of such infrastructure can be described as passing through two distinct places or as originating at a particular point and travelling in a specified way. From this concept comes the equation of a straight line, found from being given either two points or one point and a slope. Equations of lines that are parallel or perpendicular to given lines can be found. It is possible to find the distance between two points (the length of the road or storm water drain) or the middle point between two points.

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What are the features of the graph of a linear function $y = mx + c$? • Slope (m) as a rate of growth • y-intercept (c).	When slope is related to the constant adder, its role as a rate of growth becomes clear. An example is the cost per kilometre of a taxi journey. Depending on the context, the y-intercept can be interpreted to be the initial condition, such as the flag fall of a taxi ride.
How can the point where two lines intersect be found? • Solve simultaneous linear equations, graphically and algebraically • Points of intersection between two coincident straight lines • Parallel lines.	Where will two straight roads intersect? At what location will a proposed pipeline have to pass under a road? How is it possible to tell that the railway and the road are parallel or perpendicular? These questions involve the solution of a linear equation or a pair of simultaneous linear equations and the interpretation of that solution.

Sub-topic 1.2: Quadratic Relationships

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
Where do quadratic relationships arise in everyday situations?	Students construct quadratic relationships from given situations as well as examine existing models from a range of contexts. Possible contexts include areas of rectangles with fixed perimeters (including golden rectangles), business applications (profit functions), elastic collisions, and projectile paths such as shooting netball goals, voltage, and electrical power. One example is to throw a ball straight up, from 1 m above the ground with a velocity of 4 ms^{-1}. Ignoring air resistance, the height (h) in metres is $h = 1 + 4t - 5t^2$ where t is the time in seconds. (The $-5t^2$ is an approximation for $\frac{1}{2}at^2$ where a = - 9.80 m s^{-2}).
What are the features of the graph of $y = x^2$ and how are the graphs of $y = a(x - b)^2 + c$ and $y = a(x - \alpha)(x - \beta)$ related?	Features of the graphs of quadratic functions include the parabolic nature, intercepts, turning points, and axes of symmetry. Graphs are explored using technology or by drawing up tables of values.
How can quadratic expressions be rearranged algebraically so that more can learnt about their usefulness to solve problems? • Factorisation of quadratics of the form $ax^2 + bx + c$ and hence determine zeros • The quadratic formula to determine zeros • Completing the square and hence finding turning points • The discriminant and its significance for the number and nature of the zeros of a quadratic equation and the graph of a quadratic function • Using technology.	Beginning with the simpler models, students translate between the different algebraic forms of a quadratic expression. While they are doing this, there is an emphasis on the equivalence of the algebraic expressions and the information that each form provides about the model and its graph. Students use appropriate technology to examine approximate and exact solutions to quadratic equations. Students become aware of the limitations of the different techniques so that they can make an appropriate choice when looking for a solution to a quadratic equation. Positive and negative definite functions can be discussed as well as those with identical zeros. For the 'throwing the ball' example $h = 1 + 4t - 5t^2$, the maximum height and length of the time in the air can be calculated.

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What is the relationship between the solutions of a quadratic equation, the algebraic representation of the associated quadratic function, and its graph? • The sum and product of the real zeros of a quadratic equation, and the associated algebra of surds.	These relationships are crucial to the next step in analysing various situations: that is, determining (algebraically) a quadratic model to fit given data.
How can knowledge about quadratic functions be used to determine these relationships from data? • Deducing quadratic models from the zeros and one other piece of data (e.g. another point), using suitable techniques and/or technologies • Understand the role of the discriminant.	Students compare relationships determined algebraically and those found using some form of technology. Graphing technology allows a student to fit a relationship by trial and error, using the graph to determine how well it fits and also to test the uniqueness of the result. Once an appropriate quadratic relationship has been found (by whatever method), it is used to answer questions or make predictions about the situation being modelled.

Sub-topic 1.3: Inverse Proportion

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What kind of mathematical relationship describes the situation in which one variable decreases as the other increases?	The concept of the inverse relationship embodied in these functions is studied in the context of the provision of services. For example: How does the time for service vary as the number of providers increases?
What are the features of the graph of $y = \frac{1}{x}$?	How are the length and width of an envelope of a standard weight (and hence area) related? The equation and its graph in the Cartesian plane show the relationship between the two changing variables.
These graphs feature horizontal and vertical asymptotes.	Students investigate translations of the basic hyperbola in the form $y = a/(x - c)$.

Sub-topic 1.4: Cubic Polynomials

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What kinds of models have a cubic relationship? What language is used to describe the nature of the polynomial? • leading coefficient • degree.	Students explore relationships between volume and linear measure (e.g. the volume of a box created by cutting squares from the corners of a rectangular piece of card). Other places where cubic relationships arise are solubility of chemicals versus temperature, and wind speed versus power output from a wind generator.
What kinds of behaviour can be expected from the graph of a cubic function? • $y = x^3$ • $y = a(x - b)^3 + c$ • $y = a(x - \alpha)(x - \beta)(x - \gamma)$.	Students use technology to investigate the graphs of a range of cubic functions with a view to identifying the shape and the number of zeros. For example, what is the effect on the graph if the leading coefficient $a \neq 1$?
What algebraic forms can a cubic expression take? • Cubics can be written as a product of a linear and a quadratic factor or as a product of three linear factors • The significance of these forms for the shape and number of zeros of the graph.	Students use multiplication to verify the equivalence of factorised and expanded forms of cubic polynomials. Given one linear factor of a real cubic, using a quadratic with unknown coefficients, students find the other factors by inspection or by equating coefficients. Students understand what happens when $x \to \pm\infty$ The examples can be extended to look at polynomials of degree greater than 3.

Sub-topic 1.5: Functions

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What kind of equation describes a circle of which you know the radius and the location of its centre? • Equations of circles in both centre/radius and expanded form	Consider the mathematical description of the region covered by a mobile telephone tower or a pizza shop that delivers within a certain distance. Converting the equation of a circle from expanded form provides for students practice on 'completing the square'.
What are the features of the graph of $y^2 = x$ and $y = x^{\frac{1}{2}}$	Features of $y^2 = x$ include its parabolic shape and its axis of symmetry – an example of a relation that is not a function.
Understand the concept of a function (and the concept of its graph) • Domain and range • Function notation.	This includes as a mapping between sets, and as a rule or a formula that defines one variable in terms of another.
What is it possible to tell by simply looking at one or more points on a graph? • Use of function notation • Comparing points • Dependent and independent variables • Understanding the concept of a graph of a function.	The concept of dependent and independent variables (or interdependence) is considered in different contexts leading to the labelling of the axes. The use of function notation is developed. The concepts of domain and range are introduced in these contexts.
How does the graph of y = f(x) change when translated or dilated?	Explore translations for the graphs of y = f(x) + a and y = f(x+b) and dilations for the graphs of y = cf(x) and y = f(dx), considering linear, quadratic, and cubic polynomials.

Topic 2: Trigonometry (Major Topic)

The study of trigonometry enables students to solve problems drawn from contexts such as construction, design, navigation, and surveying.

The variation in demand for electricity throughout the day, the seasonal variations in climate, and the cycles within the economy are examples of periodic phenomena. Understanding how to model, and students predict trends of such phenomena.

Students learn about one particular family of periodic functions with the introduction of the basic trigonometric functions, beginning with a consideration of the unit circle, using degrees. Radian measure of angles is introduced, the graphs of the trigonometric functions are examined, and their applications in a range of settings are explored. More complex trigonometric functions are explored in Specialist Mathematics.

Sub-topic 2.1: Cosine and Sine Rules

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What tools are there for solving problems involving right-angled triangles? • Pythagoras' theorem • Trigonometric ratios.	Briefly consider right-angled triangle problems in context and with practical activities where appropriate; for example: • finding the height of an object, using a clinometer • finding the angle of inclination of the sun • determining whether or not a volleyball court is truly rectangular • calculating the length of ladder to reach an otherwise inaccessible spot • calculating the length of props needed to raise a shed wall into a vertical position.
How is it possible to solve problems in which the triangles involved are not right-angled?	Use of tools to deal with non-right triangles can be emphasised by posing problems in contexts such as surveying, building, navigation, and design. Students consider how they would find the answers to these problems, using the skills they have learnt. The validity and/or shortcomings of methods, such as scale drawing and trial and error, are discussed.
The cosine rule • Find the length of the third side when two sides and the included angle are known • Find the measure of an angle when the three sides are known	Consider the derivation of the cosine rule, using Pythagoras' theorem. Recognise that the cosine rule is a 'generalised' version of Pythagoras' theorem with a 'correction factor' for angles that are larger or smaller than $90°$. Examples using the cosine rule include the solution of contextual problems drawn from recreation and industry for an unknown side or angle.

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
The sine rule • Find the measure of an unknown angle when two sides and the non-included angle are known • Find the length of one of the unknown sides where two angles and one side are known	Justification of the sine rule by direct measurement is useful.
• Are there now sufficient tools to solve any problem involving the angles and lengths of sides of triangles?	Examples using the sine rule include the solution of contextual problems drawn from recreation and industry for an unknown side or angle. Discuss ambiguous or impossible cases, and how they arise in practical situations.
• How is it possible to find the area of a non-right triangle if the perpendicular to a side cannot be measured easily or accurately?	Derivation of the formula $A = \frac{1}{2}ab.\sin C$, using right triangles. This formula can be used to establish the sine rule.

Sub-topic 2.2: Circular Measure and Radian Measure

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What do the graphs of cos θ and sin θ look like?	As an introduction to sketching trigonometric functions, draw the graphs of cos θ and sin θ using degrees.
What is the link between the unit circle and cos θ, sin θ and tan θ in degrees? Understand the unit circle definition of cos θ, sin θ and tan θ and periodicity using degrees.	Technology is used to show this link.
Can angles be measured in different units? Define the radian measure of an angle as the length of the arc that subtends this angle at the centre of a unit circle Apply the relationship to convert between radian and degree measure Calculate lengths of arcs and areas of sectors of circle	Data generated by measuring the height of the fixed point on a bicycle wheel as it rolls can be standardised by converting the distance travelled into radius units (radians) and the height above the ground into the height above or below the axle. This is an introduction to the concept of radian measure of angles and the ‘standardised’ unit circle as a frame of reference for considering all situations that involve circular motion.

Sub-topic 2.3: Trigonometric Functions

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What is the link between the unit circle and cos θ and sin θ in radians?	Software can be used to show this link.
Understand the unit circle definition of cos θ and sin θ and periodicity using radians	Determine the exact values of cos θ and sin θ for integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$ using either unit circle or graphs.
What function best describes the horizontal and vertical position of a point moving round a circle? • The functions y = sin x and y=cos x	Draw the graph of y = sin x using radians and determine and describe some key properties.
• Recognise changes in amplitude, period, and phase	Using graphing technology, students explore the effects of the three control numbers on transforming the graph of y = sin x and y = cos x. Examine the graphs of • y = Asin x and y = Acos x • y = sin Bx and y = cos Bx • y = sin(x + C) and y = cos(x + C).
• Identify contexts suitable for modelling by trigonometric functions and use them to solve practical problems.	Students sketch the graphs of simple sinusoidal functions, to represent a range of different contexts.
• Solving trigonometric equations using technology and algebraically in simple cases	Students solve these equations both graphically (using technology) and algebraically for simple cases such as cos x = $\frac{1}{2}$ or sin 2x = $\frac{1}{2}$
What special relationships can be observed by examining the sine and cosine functions and their behaviour in the unit circle? • $\sin(-x) = -\sin x$ • $\cos(-x) = \cos x$ • $\sin(x + \frac{\pi}{2}) = \cos x$ • $\cos(x - \frac{\pi}{2}) = \sin x$	Students consider the deduction of these useful identities by looking at the unit circle. They compare their graphs with those of other students to recognise, for example, change in the amplitude. A more extensive exploration of trigonometric identities is included in Specialist Mathematics.

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
Where does the tangent function fit into all this? • Understand the relationship between the angle of inclination and the gradient of the line • $\tan x = \frac{\sin x}{\cos x}$ • The graphs of the functions o y = tan x o y = tan Bx o y = tan(x + C).	The slope of the radius *OP* as *P* travels round the unit circle and generates the tangent function. Students investigate the behaviour of this function and its graph with a view to understanding ways in which it is different from, and similar to, the sinusoidal functions. Students' awareness of the construction that gives this function its name, as this offers another way of understanding the behaviour of the function for values between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. Determine the exact values of tan θ for integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$.

Topic 3: Counting and Probability (Minor Topic)

The study of inferential statistics begins in this unit with the introduction to counting techniques. Students build on their understanding of the fundamentals of probability and are introduced to the concepts of conditional probability and independence.

Sub-topic 3.1: Combinatorics

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How can the number of ways something will occur be counted without listing all of the outcomes?	Calculations can sometimes involve working out the number of different ways in which something can happen. Since simply listing the ways can be tedious and unreliable, it is helpful to work out some techniques for doing this kind of counting.
How can the number of ways of making several different choices in succession be counted? • The multiplication principle • Factorials and factorial notation • Permutations	Students explore the multiplication principle by using tree diagrams and tables. For example, finding the number of ways in which a three course meal can be chosen from a menu. By looking at the number of arrangements, students can find the number of ways of arranging *n* different things is *n*!, leading to the number of permutations (or arrangements) of *n* objects taking *r* at a time $P^n_r = n(n-1)(n-2)\ldots(n-r+1)$ $= \frac{n!}{(n-r)!}$ Students solve problems involving the multiplication principle and permutations (using only discrete objects). For example, students determine the number of possible car number plates or the number of different ways in which 5 candidates in an election can be listed on a ballot paper.

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How can the number selections be counted for different groups? • Understand the notion of a combination as an unordered set of distinct objects	Students work with ordered arrangements and unordered selections and use the multiplication principle to develop the link between the number of ordered arrangements and the number of unordered selections $\boldsymbol{r}! \times \boldsymbol{number\ of\ unordered\ selections} = \boldsymbol{P^{n}_{r}}$ Examples include counting the number of handshakes for a group of people, or the number of teams of 3 students that can be chosen from the 5 students who have nominated themselves.
• The number of combinations (or selections) of r objects taken from a set of n distinct objects is C^n_r	To find the number of combinations (or selections), first count the number of permutations (or arrangements) and then divide by the number of ways in which the objects can be arranged.
• Use $\boldsymbol{C^{n}_{r}} = \frac{\boldsymbol{P^{n}_{r}}}{\boldsymbol{r}!} = \frac{\boldsymbol{n}!}{(\boldsymbol{n}-\boldsymbol{r})!\boldsymbol{r}!}$ to solve problems	For example, how many cricket teams of 11 players can be chosen from a squad of 14 players or if an airline has 10 passengers on standby, how many ways could the airline choose the passengers to fill the remaining 4 seats.
Use the notation $\binom{n}{r}$ and the formula $\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$ for the number of combinations of r objects taken from a set of n distinct objects .	Alternatively, $\boldsymbol{C^{n}_{r}} = \binom{\boldsymbol{n}}{\boldsymbol{r}} = \frac{\boldsymbol{n(n-1)(n-2)\ldots.(n-r+1)}}{\boldsymbol{r(r-1)(r-2)\ldots.1}}$ Students use technology for calculating with larger numbers.
How are combinations related to the coefficients in the expansion of $(x + y)^n$? • Expand $(x + y)^n$ for integers n = 1,2,3,4 • Recognise the numbers $\binom{n}{r}$ as binomial coefficients, (as coefficients in the expansion of $(x+y)^n$).	It is useful to start with expanding (1+a)= (1+a)(1+b)= (1+a)(1+b)(1+c)= (1+a)(1+b)(1+c)(1+d)= Organise the terms in order of the number of factors then make x = a = b = c = d to link with the notion of the coefficients in the expansion of $(1 + x)^n$ as the number of combinations (unordered sets of objects).
Is there a pattern connecting the values of $\binom{n}{r}$?	Students generate Pascal's triangle, building each row using addition of elements in the previous row. Use properties of Pascal's triangle to highlight the properties of $\binom{n}{r}$.

Sub-topic 3.2: Fundamentals of Probability

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How is it possible to decide and describe how likely it is that certain events will occur?	Probability is a measure of the likelihood of occurrence of an event.
	Consider the probability scale: $0 \le P(A) \le 1$ for each event A, with $P(A)=0$ if A is an impossibility and $P(A)=1$ if A is a certainty.
	Reinforce the concepts and language of outcomes, sample spaces, and events as sets of outcomes.
	Use relative frequencies obtained from data as point estimates of probabilities. For example, calculate the relative frequency (experimental probability) of rolling a 2 on a dice over 10, 20, 30, …200 trials. Graph the experimental probability versus the number of trials, and describe the trend.

Sub-topic 3.3: Conditional Probability and Independence

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How can the probability of successive events using tree diagrams and multiplication principle be determined?	In many applications of probability the chances that an event will happen depends on more than one factor. For instance, a car insurance premium can depend on a driver's age, gender, and driving record. In medical testing the probability of an outcome depends on the chances that a person has the condition as well as on the reliability of the test itself. In other applications there is a succession of events or trials (e.g. travelling to work through several sets of traffic lights).
• Independent events and dependent events	Examples illustrate the difference between successive tosses of a coin (where the probabilities of successive outcomes do not depend on previous outcomes) and the drawing of names from a hat without replacement (where probabilities depend on previous outcomes). In cases where the probabilities of successive outcomes are independent of one another, the tree diagram can often be dispensed with, once students understand the multiplication principle, i.e. $P(A \cap B) = P(A).P(B)$. Tree diagrams are a useful start when the probability of one event depends on the outcome of a prior event leading to $P(A \cap B) = P(A).P(B\|A)$. If A and B are independent events then $P(B\|A) = P(B)$.

Topic 4: Statistics (Minor Topic)

An exploration of distributions and measures of spread, extending students' knowledge of the measures of central tendency in statistics, provides the background required for the study of inferential statistics in Stage 2.

Sub-topic 4.1: Discrete and Continuous Random Data

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How are discrete variables different to continuous variables? • Continuous variables may take any value (often within set limits), examples include height and mass • Discrete variables may only take specific values; examples include the number of eggs that can be purchased at a supermarket.	Activities such as: • measuring the length of all the new pencils in a box • measuring the actual mass of different '1 kg' bags of potatoes illustrate that values that may be considered as fixed are actually variable, with random values. Students measure different variables (their height, mass, number of teeth, etc.) and classify them as discrete or continuous. Low resolution measuring devices (e.g. rulers that only measure down to centimetres) can be used to show that continuous variables may be recorded in a way that makes them appear to be discrete.

Sub-topic 4.2: Samples and Statistical Measures

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What values are useful in describing the centre of a sample of data? • Briefly consider mean, median, and mode.	Comparisons of different data sets can show the strengths and weaknesses of the different central tendencies.
What values are useful in describing the spread of a sample of data? • Consider range and interquartile range. • Standard deviation of a sample gives a useful measure of spread, which has the same units as the data.	Comparisons of different data sets can also show the strengths and weaknesses of the different measures of spread. Calculations of standard deviations of small samples using the formula: $s = \sqrt{\frac{1}{N-1}\sum(x-\bar{x})^2}$ illustrate what it calculates, and can be replaced with calculations using electronic technology once the concept is understood.

Sub-topic 4.3: Normal Distributions

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
Why do normal distributions occur? • Value of the quantity is the combined effect of a number of random errors.	Students are introduced to a variety of quantities whose variation is approximately normal (e.g. the volume of a can of soft drink or the lifetime of batteries). In investigating why normal distributions occur, students build a spreadsheet that generates a large amount of data from the sum of a finite number of random numbers.
What are the features of normal distributions? • Bell-shaped • Position of the mean • Symmetry about the mean • Characteristic spread • Unique position of one standard deviation from the mean.	A refined spreadsheet from this activity allows students to see the features of normal distributions develop (i.e. whatever the mean or standard deviation, all normal distributions have approximately 68% of the data one standard deviation on either side of the mean; approximately 95% is within 2 standard deviations; and approximately 99.7% is within 3 standard deviations).
Why are normal distributions so important? • Variation in many quantities occurs in an approximately normal manner, and can be modelled using a normal distribution How can the percentage of a population meeting a certain criterion be estimated for normal distributions?	Students calculate proportions or probabilities of occurrences within plus or minus integer multiples of standard deviations of the mean.

Topic 5: Growth and Decay (Major Topic)

This topic covers the study of exponential and logarithmic functions under the unifying idea of modelling growth. The mathematical models investigated arise from actual growth situations. By developing and applying these mathematical models, students see how the wider community might use them for analysis, prediction, and planning.

So that actual data can be handled efficiently, technology is used extensively in this topic, for both graphing and calculation. Much of the technology has the facility to fit curves to data automatically. This allows students to compare their own models with a solution from another source.

Sub-topic 5.1: Indices and Index Laws

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How can indices be used to solve problems?	Knowledge of indices enables students to consider exponential function and gain an appreciation of how exponential functions can model actual situations involving growth and decay. Briefly consider indices (including negative and fractional indices) and the index laws. Simplify algebraic products and quotients using index laws, applying knowledge of index laws to algebraic terms, and simplifying algebraic expressions using positive and negative integral indices, and fractional indices. Use radicals and convert to and from fractional indices. Irrational numbers occur when using the quadratic formula. Define rational and irrational numbers and perform operations with surds and fractional indices • understanding that the real number system includes irrational numbers • extending the index laws to rational number indices • performing the four operations with surds.

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How can very large and very small numbers be managed? Understand and use scientific notation and significant figures.	Express very large and very small numbers in scientific notation, and numbers expressed in scientific notation as whole numbers or decimals. Calculators give large and small answers in scientific notation to a degree of accuracy that is often not required or is misleading. Significant figures are used to round off numbers to indicate an appropriate degree of precision.

Sub-topic 5.2: Exponential Functions

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What is meant by an exponential relationship? Establish and use the algebraic properties of exponential functions.	Some examples are compound interest, depreciation, half-lives of radioactive material, simple population models (bacteria, locusts, etc.).
What kind of behaviour do exponential functions show? Recognise the qualitative features of the graph of $y = a^x$ $(a>0)$ and of its translations $y = a^x + b$ and $y = a^{x+c}$ and dilation $y = ka^x$.	Graphs of exponential functions involving powers of simple numbers such as 2, 10, and ½ should be examined here (although others can be used). Technology can be used to explore different graphs. The similarities that characterise these functions, such as asymptotes, intercepts, and behavior as $x \to \pm\infty$, are emphasised.
How can problems that involve exponential functions be solved?	Determining the x-value for a given y-value (e.g. finding when a population should reach a certain value or finding the doubling time) can be done graphically, using technology to refine the answers, and algebraically in simple cases.

Sub-topic 5.3: Logarithmic Functions

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How is it possible to get an exact solution to an equation where the power is the unknown quantity? • Definition of the logarithm of a number • Rules for operating with logarithms $\log_a b = x \Leftrightarrow a^x = b$ and the relationships • $\log_a a^x = a^{\log_a x} = x$ • $\log_a mn = \log_a m + \log_a n$ • $\log_a \frac{m}{n} = \log_a m - \log_a n$ • $\log_a b^m = m \log_a b$	The study of logarithms arises from the need to be able to find an exact mathematical solution to exponential equations. Discussion of the historical development of the technique, and its power in enabling mathematicians to solve a range of problems, is useful and interesting.
• Solving exponential equations, using logarithms (base 10).	Problems involving determining the x-value for a given y-value in equations of the form $y = a^x$ (e.g. finding when a population should reach a certain value or finding the doubling time) is revisited and performed using logarithms (base 10). New problems posed in context to reinforce the necessary skills. Students explore the 'rule of 72' to determine the 'doubling time' given the percentage rate of growth. For example, if the population is growing at 6% per annum, then the population will double in 12 years.

Topic 6: Introduction to Differential Calculus (Major Topic)

Rates and average rates of change are introduced, followed by the key concept of the derivative as an 'instantaneous rate of change'. These concepts are reinforced numerically, by calculating difference quotients both geometrically, as slopes of chords and tangents, and algebraically. Calculus is developed to study the derivatives of polynomial functions, with simple applications of the derivative to curve sketching, calculating slopes and equations of tangents, determining instantaneous velocities, and solving optimisation problems.

Sub-topic 6.1: Rate of Change

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What is a rate of change? • A rate of change is a ratio of the change in one quantity compared with that in a second related quantity. How can the rate of change of a non-linear function $f(x)$ over an interval be considered? • The average rate of change of function $f(x)$ in the interval from a to b is $\frac{f(b)-f(a)}{b-a}$ • The average rate of change of function $f(a)$ in the interval from a to $a+h$ is $\frac{f(a+h)-f(a)}{h}$ • Use the notation $\frac{\delta y}{\delta x}$ for $\frac{f(x+h)-f(x)}{h}$ where $y=f(x)$ • The average rate of change interpreted as the slope of a chord.	This concept can be covered in the context of, for example, finding average speeds, costs per kilogram, litres of water used per day, or watts of power used per day. A constant rate of change can be identified through the exploration of examples such as running, driving at a steady rate, or leaving a mains electrical appliance operating for a period of time. This can be discovered • numerically in a table with a constant adder • algebraically as a property of a linear function • graphically as the gradient of a straight line. Using some of the contexts already mentioned, the concept of non-constant rate of change can be explored by considering average rates over different time intervals, for example: • an accelerating car • water delivered from a cask or dispenser (under gravity) • power delivered from a storage battery for a sufficiently long time to flatten the battery.

Sub-topic 6.2: The Concept of a Derivative

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
	This concept can be strengthened by working: • numerically from tables of data • algebraically from a formula • graphically (and geometrically) by considering gradients of chords across graphs of curves (graphics calculators, interactive geometry, and graphing software provide invaluable visual support, immediacy, and relevance for this concept). Applying all three approaches in one context strengthens the presentation of this concept. To aid progression to future subtopics, students explore how the average rate of change varies as the width of the interval decreases.
How can the rate of change at a point be approximated?	The rate of change across an interval is an approximation of the rate of change at a point (instantaneous rate of change). As the interval decreases, the approximation approaches the instantaneous rate of change (also to be interpreted as a chord approaching a tangent).
What is a limit?	The instantaneous rate of change of a function at a point is the limit of the average rate of change over an interval that is approaching zero. The notion of a limit can be developed by considering graphs of the form $g(x)=\frac{x^2-4}{x-2}$, or similar. Alternatively, the notion of a limit can be developed by attempting to evaluate the fraction $\frac{(a+h)^2-a^2}{h}$ as h approaches zero.
How can the instantaneous rate of change using derivatives be determined from first principles?	The derivative can be introduced as a summary of the concepts of rates of change and as a way to calculate an instantaneous rate of change.

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
Find the derivative function from first principles using $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$.	From first principles, find • the derivative at a given point $f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ and/or $f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. $f'(x)=\frac{dy}{dx}=\lim_{\delta x\to 0}\frac{\delta y}{\delta x}$ • the derivatives of functions such as $f(x) = x^2 - 6x$ at particular points and as functions. • the derivative of x^n for integer values of n as an introduction to the development of the rules of differentiation.

Sub-topic 6.3: Computations of Derivatives

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How can the derivative of x^n, where n is a real constant, be found?	Estimate numerically the value of a derivative, for simple power functions. Establish the formula $\frac{dy}{dx}=nx^{n-1}$ when $y = x^n$ by expanding $(x + h)^n$.

Sub-topic 6.4: Properties of Derivatives

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
Is the derivative a function?	Briefly consider the definition of a function.
What are the rules that apply to differentiation? Recognise and use the linearity of the derivative. Calculate derivatives of polynomials and other linear combinations of power functions.	The use of differentiation by first principles for a number of examples of simple polynomials develops the rule $h'(x) = f'(x) \pm g'(x)$. for $h(x) = f(x) \pm g(x)$ which leads to $h'(x) = kf'(x)$ for $h(x) = kf(x)$

Sub-topic 6.5: Applications of Derivatives

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How can differentiation be used to solve problems? Solve problems that use polynomials involving the following concepts: • the slope and equation of a tangent • displacement and velocity • rates of change – increasing and decreasing functions	Students use functional models and their derivatives in the given contexts. Focus on position versus time graphs to describe motion where the velocity equates to the slope of the tangent at any point on the graph. Use a sign diagram to determine intervals in which the function is increasing or decreasing.
• maxima and minima, local and global – stationary points – sign diagram of the first derivative – end points	Use displacement functions and their first derivatives: object changes direction when velocity changes sign; object is at rest when velocity is zero.
• optimisation.	Examine optimisation problems involving simple polynomials to describe polynomials and relationships between perimeter and area, area, and volume to minimise costs, optimal dimensions of 3D objects. The following are examples of possible contexts: • economics • population dynamics • energy consumption • water use • drug concentration.

ASSESSMENT SCOPE AND REQUIREMENTS

Assessment at Stage 1 is school based

EVIDENCE OF LEARNING

The following assessment types enable students to demonstrate their learning in Stage 1 Mathematical Methods:

Assessment Type 1: Skills and Applications Tasks
Assessment Type 2: Mathematical Investigation.

For a 10-credit subject, students should provide evidence of their learning through four assessments. Each assessment type should have a weighting of at least 20%.

Students complete:

- at least two skills and applications tasks
- at least one mathematical investigation.

For a 20-credit subject, students should provide evidence of their learning through eight assessments. Each assessment type should have a weighting of at least 20%.

Students complete:

- at least four skills and applications tasks
- at least two mathematical investigations.

It is anticipated that from 2018 all assessments will be submitted electronically.

ASSESSMENT DESIGN CRITERIA

The assessment design criteria are based on the learning requirements and are used by teachers to:

- clarify for the student what he or she needs to learn
- design opportunities for the student to provide evidence of his or her learning at the highest level of achievement.

The assessment design criteria consist of specific features that:

- students need to demonstrate in their evidence of learning
- teachers look for as evidence that students have met the learning requirements.

For this subject, the assessment design criteria are:

- concepts and techniques
- reasoning and communication.

The specific features of these criteria are described below.

The set of assessments, as a whole give students opportunities to demonstrate each of the specific features by the completion of study of the subject.

Concepts and Techniques

The specific features are as follows:

CT1 Knowledge and understanding of concepts and relationships

CT2 Selection and application of techniques and algorithms to solve problems in a variety of contexts

CT3 Development and application of mathematical models

CT4 Use of electronic technology to find solutions to mathematical problems.

Reasoning and Communication

The specific features are as follows:

RC1 Interpretation and evaluation of mathematical results, with an understanding of their reasonableness and limitations

RC2 Knowledge and use of appropriate mathematical notation, representations, and terminology

RC3 Communication of mathematical ideas and reasoning, to develop logical arguments

RC4 Development and testing of valid conjectures.

SCHOOL ASSESSMENT

Assessment Type 1: Skills and Applications Tasks

For a 10-credit subject, students complete at least two skills and applications tasks.

For a 20-credit subject, students complete at least four skills and applications tasks.

Skills and applications tasks are completed under the direct supervision of the teacher.

Students find solutions to mathematical problems that may:

- be routine, analytical, and/or interpretative
- be posed in a variety of familiar and new contexts
- require discerning use of electronic technology.

In setting skills and applications tasks, teachers may provide students with information in written form or in the form of numerical data, diagrams, tables, or graphs. A task should require students to demonstrate an understanding of relevant mathematical concepts and relationships.

Students select appropriate techniques or algorithms and relevant mathematical information to find solutions to routine, analytical, and/or interpretative problems.

Students provide explanations and arguments, and use correct mathematical notation, terminology, and representation throughout the task. They may be required to use electronic technology appropriately to aid and enhance arriving at the solution of some problems.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:

- concepts and techniques
- reasoning and communication.

Assessment Type 2: Mathematical Investigation

For a 10-credit subject, students complete at least one mathematical investigation.

For a 20-credit subject, students complete at least two mathematical investigations.

Students investigate mathematical relationships, concepts, or problems, which may be set in an applied context. The subject of a mathematical investigation may be derived from one or more subtopics, although it can also relate to a whole topic or across topics.

A mathematical investigation may be initiated by a student, a group of students, or the teacher. Teachers may give students a clear, detailed, and sequential set of instructions for part of the investigation or to initiate the investigation, or may provide guidelines for students to develop contexts, themes, or aspects of their own choice. Teachers should give some direction about the appropriateness of each student’s choice, and guide and support students’ progress in a mathematical investigation.

A mathematical investigation may provide an opportunity for students to work collaboratively to achieve the learning requirements. If an investigation is undertaken by a group, students explore the problem and gather data together to develop a model or solution individually. Each student must submit an individual report.

Teachers may need to provide support and clear directions for the first mathematical investigation. Where students undertake more than one investigation, subsequent investigations could be less directed and set within more open-ended contexts.

Students demonstrate their problem-solving strategies as well as their knowledge, skills, and understanding in the investigation. They are encouraged to use a variety of mathematical and other software (e.g. Computer Algebra Systems, spreadsheets, statistical packages) to assist in their investigation. The generation of data and the exploration of patterns and structures, or changing parameters, may provide an important focus. From these, students may recognise different patterns or structures. Notation, terminology, forms of representation of information gathered or produced, calculations, technological skills, and results are important considerations.

Students complete a report on the mathematical investigation.

In the report, they formulate and test conjectures, interpret and justify results, draw conclusions, and give appropriate explanations and arguments.

The report may take a variety of forms, but would usually include the following:

- an outline of the problem and context
- the method required to find a solution, in terms of the mathematical model or strategy used
- the application of the mathematical model or strategy, including:
  - relevant data and/or information
  - mathematical calculations and results, using appropriate representations
  - the analysis and interpretation of results, including consideration of the reasonableness and limitations of the results
- the results and conclusions in the context of the problem
- a bibliography and appendices, as appropriate.

The format of an investigation report may be written or multimodal.

Each investigation report should be a maximum of 8 pages if written, or the equivalent in multimodal form.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:

- concepts and techniques
- reasoning and communication.

PERFORMANCE STANDARDS

The performance standards describe five levels of achievement, A to E.

Each level of achievement describes the knowledge, skills and understanding that teachers refer to in deciding how well a student has demonstrated his or her learning, on the basis of the evidence provided.

During the teaching and learning program the teacher gives students feedback on their learning, with reference to the performance standards.

At the student's completion of study of a subject, the teacher makes a decision about the quality of the student's learning by:

- referring to the performance standards
- taking into account the weighting given to each assessment type
- assigning a subject grade between A and E.

Performance Standards for Stage 1 Mathematical Methods

	Concepts and Techniques	Reasoning and Communication
A	Comprehensive knowledge and understanding of concepts and relationships. Highly effective selection and application of techniques and algorithms to find efficient and accurate solutions to routine and complex problems in a variety of contexts. Successful development and application of mathematical models to find concise and accurate solutions. Appropriate and effective use of electronic technology to find accurate solutions to routine and complex problems.	Comprehensive interpretation and critical evaluation of mathematical results, with an in-depth understanding of their reasonableness and possible limitations. Proficient and accurate use of appropriate mathematical notation, representations, and terminology. Highly effective communication of mathematical ideas and reasoning to develop logical and concise arguments. Effective development and testing of valid conjectures.
B	Some depth of knowledge and understanding of concepts and relationships. Mostly effective selection and use of mathematical techniques and algorithms to find accurate solutions to routine and some complex problems in a variety of contexts. Mostly successful development and application of mathematical models to find accurate solutions. Mostly appropriate and effective use of electronic technology to find accurate solutions to routine and some complex problems.	Mostly appropriate interpretation and some critical evaluation of mathematical results, with some depth of understanding of their reasonableness and possible limitations. Mostly accurate use of appropriate mathematical notation, representations, and terminology. Mostly effective communication of mathematical ideas and reasoning to develop logical arguments. Mostly effective development and testing of valid conjectures.
C	Generally competent knowledge and understanding of concepts and relationships. Generally effective selection and use of mathematical techniques and algorithms to find mostly accurate solutions to routine problems in a variety of contexts. Some development and successful application of mathematical models to find generally accurate solutions. Generally appropriate and effective use of electronic technology to find mostly accurate solutions to routine problems.	Generally appropriate interpretation and evaluation of mathematical results, with some understanding of their reasonableness and possible limitations. Generally appropriate use of mathematical notation, representations, and terminology, with some inaccuracies. Generally effective communication of mathematical ideas and reasoning to develop some logical arguments. Development and testing of generally valid conjectures.
D	Basic knowledge and some understanding of concepts and relationships. Some effective selection and use of mathematical techniques and algorithms to find some accurate solutions to routine problems in some contexts. Some application of mathematical models to find some accurate or partially accurate solutions. Some appropriate use of electronic technology to find some accurate solutions to routine problems.	Some interpretation and attempted evaluation of mathematical results, with some awareness of their reasonableness and possible limitations. Some appropriate use of mathematical notation, representations, and terminology, with some accuracy. Some communication of mathematical ideas, with attempted reasoning and/or arguments. Attempted development or testing of a reasonable conjecture.
E	Limited knowledge or understanding of concepts and relationships. Attempted selection and limited use of mathematical techniques or algorithms, with limited accuracy in solving routine problems. Attempted application of mathematical models, with limited accuracy. Attempted use of electronic technology, with limited accuracy in solving routine problems.	Limited interpretation or evaluation of mathematical results, with limited awareness of their reasonableness and possible limitations. Limited use of appropriate mathematical notation, representations, or terminology, with limited accuracy. Attempted communication of mathematical ideas, with limited reasoning. Limited attempt to develop or test a conjecture.

ASSESSMENT INTEGRITY

The SACE Assuring Assessment Integrity Policy outlines the principles and processes that teachers and assessors follow to assure the integrity of student assessments. This policy is available on the SACE website (wwww.sace.sa.edu.au) as part of the SACE Policy Framework.

The SACE Board uses a range of quality assurance processes so that the grades awarded for student achievement in the school assessment are applied consistently and fairly against the performance standards for a subject, and are comparable across all schools.

Information and guidelines on quality assurance in assessment at Stage 1 are available on the SACE website (www.sace.sa.edu.au).

SUPPORT MATERIALS

SUBJECT-SPECIFIC ADVICE

Online support materials are provided for each subject and updated regularly on the SACE website (www.sace.sa.gov.au) Examples of support materials are sample learning and assessment plans, annotated assessment tasks, annotated student responses, and recommended resource materials.

ADVICE ON ETHICAL STUDY AND RESEARCH

Advice for students and teachers on ethical study and research practices is available in the guidelines on the ethical conduct of research in the SACE on the SACE website. (www.sace.sa.edu.au).

Stage 2
Mathematical Methods

LEARNING SCOPE AND REQUIREMENTS

LEARNING REQUIREMENTS

The learning requirements summarise the key skills, knowledge and understanding that students are expected to develop and demonstrate through learning in Stage 2 Mathematical Methods.

In this subject, students are expected to:

1. understand mathematical concepts, demonstrate mathematical skills, and apply mathematical techniques
2. investigate and analyse mathematical information in a variety of contexts
3. think mathematically by posing questions and solving problems; developing and evaluating models; making, testing, and proving conjectures
4. interpret results, draw conclusions, and determine the reasonableness of solutions in context
5. make discerning use of electronic technology to solve problems and to refine and extend mathematical knowledge
6. communicate mathematically and present mathematical information in a variety of ways.

CONTENT

Stage 2 Mathematical Methods is a 20-credit subject.

Stage 2 Mathematical Methods focuses on the development of mathematical skills and techniques that enable students to explore, describe, and explain aspects of the world around them in a mathematical way. It places mathematics in relevant contexts and deals with relevant phenomena from the students' common experiences, as well as from scientific, professional, and social contexts.

The coherence of the subject comes from its focus on the use of mathematics to model practical situations, and on its usefulness in such situations. Modelling, which links the two mathematical areas to be studied, Calculus and Statistics, is made more practicable by the use of electronic technology.

The ability to solve problems based on a range of applications is a vital part of mathematics in this subject. As both calculus and statistics are widely applicable as models of the world around us, there is ample opportunity for problem solving throughout this subject.

Stage 2 Mathematical Methods consists of the following list of six topics:

- Topic 1: Further Differentiation and Applications
- Topic 2: Discrete Random Variables
- Topic 3: Integral Calculus
- Topic 4: Logarithmic Functions
- Topic 5: Continuous Random Variables and the Normal Distribution
- Topic 6: Sampling and Confidence Intervals.

The suggested order of the topics is a guide only; however, students study all six topics. If Mathematical Methods is to be studied in conjunction with Specialist Mathematics, consideration should be given to appropriate sequencing of the topics across the two subjects.

Each topic consists of a number of subtopics. These are presented in the subject outline in two columns as a series of key questions and key concepts side by side with considerations for developing teaching and learning strategies.

The ‘key questions and key concepts’ cover the prescribed areas for teaching, learning, and assessment in this subject. The ‘considerations for developing teaching and learning strategies’ are provided as a guide only.

A problem-based approach is integral to the development of the mathematical models and associated key concepts in each topic. Through key questions teachers can develop the key concepts and processes that relate to the mathematical models required to address the problems posed.

The ‘considerations for developing teaching and learning strategies’ present problems for consideration and guidelines for sequencing the development of the concepts. They also give an indication of the depth of treatment and emphases required.

Although the material for the external examination will be based on the ‘key questions and key concepts’ outlined in the five topics, the ‘considerations for developing teaching and learning strategies’ may provide useful contexts for examination questions.

Students should have access to technology, where appropriate, to support the computational aspects of these topics.

Calculus

The following three topics relate to the study of Calculus:

- Topic 1: Further Differentiation and Applications
- Topic 3: Integral Calculus
- Topic 4: Logarithmic Functions

Calculus is essential for developing an understanding of the physical world as many of the laws of science are relationships involving rates of change. The study of calculus provides a basis for understanding rates of change in the physical world, and includes the use of functions, their derivatives and integrals, in modelling physical processes.

In this area of study, students gain a conceptual grasp of introductory calculus, and the ability to use its techniques in applications. This is achieved by working with various kinds of mathematical models in different situations, which provide a context for investigating and analysing the mathematical function behind the mathematical model.

The study of calculus continues from Stage 1 with the derivatives of exponential, logarithmic, and trigonometric functions and their applications, together with differentiation techniques and applications to optimisation problems and graph sketching. It concludes with integration, both as a process that reverses differentiation and as a way of calculating areas. The fundamental theorem of calculus as a link between differentiation and integration is emphasised.

Statistics

The following three topics relate to the study of Statistics:

- Topic 2: Discrete Random Variables
- Topic 5: Continuous Random Variables and the Normal Distribution
- Topic 6: Sampling and Confidence Intervals

Statistics are used to describe and analyse phenomena involving uncertainty and variation. The study of statistics enables students to describe and analyse phenomena that involve uncertainty and variation. In this area of study, students move from asking statistically sound questions towards a basic understanding of how and why statistical decisions are made. The area of study provides students with opportunities and techniques to examine argument and conjecture from a ‘statistical’ point of view. This involves working with discrete and continuous variables, and the normal distribution in a variety of contexts; discovering and using the power of the central limit theorem; and understanding the importance of this theorem in statistical decision-making.

Topic 1: Further Differentiation and Applications

Sub-topic 1.1: Introductory Differential Calculus

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What types of problems can be solved by finding the derivatives of polynomials? • The derivative of a polynomial can be used to find the slope of tangents to the polynomials	This is an extension of the topic ‘Introduction to Differential Calculus’ from Stage 1 Mathematical Methods.
• When an object’s displacement is described by a polynomial function, the derivative can be used to find the instantaneous velocity • The derivative of a polynomial function can be used to determine the rate of change, the position of any local maxima or minima, and when the function is increasing or decreasing.	Polynomials and linear combinations of power functions can be used to model many scenarios, such as areas of quadrilaterals, volumes of boxes, numbers of people at an event, and the speed of objects. These applications of calculus are included here so that conceptual understanding is built with functions that students have differentiated in Stage 1. As further differentiation skills are developed, these applications are revisited.

Sub-topic 1.2: Differentiation Rules

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What are the different algebraic structures of functions? • Functions can be classified as sums, products or quotients of simpler functions through an analysis of the use of grouping symbols (brackets) and a knowledge of the order of operations.	Correct identification of the algebraic structure of a function allows the correct differentiation process to be applied.
How is it possible to differentiate composite functions of the form $h(x)=f(g(x))$, with at most one application of the chain rule? $h(x)=f(g(x))$ has a derivative $$h'(x)=f'(g(x))g'(x)$$ This is called the Chain Rule.	The chain rule can be verified for a number of simple examples by establishing: numerical results for the function's derivative, using electronic technology the same results algebraically by evaluating the product of the two factors proposed by the chain rule. An algebraic approach would be to compare applying the chain rule and simplifying, with simplifying and differentiating term by term.
How do we differentiate the product of two functions $h(x)=f(x).g(x)$? $h(x)=f(x).g(x)$ has a derivative $h'(x)=f'(x)g(x)+f(x)g'(x)$ This is called the Product Rule.	Methods similar to those used to introduce the chain rule can be used for the product rule. A more general justification follows, allowing progression to using the rule when the alternative expansions are too cumbersome or not available.
How do we differentiate functions of the form $h(x)=\frac{f(x)}{g(x)}$? • $h(x)=\frac{f(x)}{g(x)}$ has a derivative $h'(x)=\frac{f'(x)g(x)-f(x)g'(x)}{[g(x)]^2}$ This is called the Quotient Rule	The quotient rule can be established by applying the product rule to $$f(x)=h(x)(g(x))^{-1}$$

Sub-topic 1.3: Exponential Functions

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What form does the derivative function of an exponential function $y = ab^x$ take? • The derivative of $y = ab^x$ is a multiple of the original function.	Working in the context of a population with a simple doubling rule $P(t) = 2^t$ and using electronic technology, it is possible to show that the derivative seems to be a multiple of the original growth function. This leads to a number of 'What if …' questions • What if the derivative is investigated numerically by examining $\frac{2^{t+h} - 2^t}{h} = \frac{2^h - 1}{h}.2^t$ for smaller and smaller values of h? • What if the function considered is $P(t) = 5^t$? • What if a base could be found that provided for $P'(t) = P(t)$?
For what value of *b* is the derivative equal to the original function $y = b^x$? • There exists an irrational number *e* so that $\frac{dy}{dx} = y = e^x$. The approximate value of *e* is 2.7182818	Electronic technology can be used to graph both the original function and the derivative on the same axes. Changing the value of b from 2 to 3, to 2.8 to 2.7 to 2.72 can lead to the conjecture that there is a base close to 2.72 for which the original function and the derivative are the same function. Numerical and graphical investigations of the expression $\left(1+\frac{1}{t}\right)^t$ also lead to the existence of the number conventionally called e.
What are the derivatives of $y = e^x$ and $y = e^{f(x)}$? • The chain rule can be used to show that the derivative of $y = e^{f(x)}$ is given by $\frac{dy}{dx} = e^{f(x)} f'(x)$	The chain rule, product rule, and quotient rule are considered with the inclusion of e^x and $e^{f(x)}$.

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What do the graphs of exponential functions look like? • Many exponential functions show growth or decay. This growth/decay may be unlimited or asymptotic to specific values.	Electronic technology can be used to explore the different features of various types of exponential graphs. Suitable functions for exploration include the surge function $f(x) = axe^{-bx}$ and the logistic function $P(t)=\dfrac{L}{1+Ae^{-bt}}$. Exploration of the location of the turning point of $y = \dfrac{x+a}{e^x}$ can provide an opportunity for the development, testing, and proof of conjectures.
What types of problems can be solved by finding the derivatives of exponential functions? Derivatives can be used to: • find the slope of tangents to the graphs of functions. • find the instantaneous velocity, when an object's displacement is described by an exponential function • determine the rate of change, the position of any local maxima or minima, and when the function is increasing or decreasing.	These applications of calculus have been developed earlier, using polynomials and linear combinations of power functions. The use of e^x and $e^{f(x)}$ within the modelling function shows the need for a way to determine the exact value of x such that e^x is equal to specific values. This is covered in the 'Logarithmic Functions' topic. For some examples, approximate values can be determined using electronic technology.
Use exponential functions and their derivatives to solve practical problems where exponential functions model actual examples.	Exponential functions can be used to model many actual scenarios, including those involving growth and decay.

Sub-topic 1.4: Trigonometric functions

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How can cosine and sine be considered as functions? • Graphing sine and cosine functions in the Stage 1 topic ‘Trigonometry’ introduced the concept of radian angle measure and the use of sine and cosine to define different aspects of the position of a moving point. • When t is a variable measured in radians (often time) $\sin t$ and $\cos t$ are periodic functions.	Students reinforce their understanding of cosine and sine in terms of the circular motion model with an emphasis on the coordinates of the moving point as functions of time, t.
What are the derivatives of $y = \sin t$ and $y = \cos t$? • $y = \sin t$ has a derivative $\frac{dy}{dt} = \cos t$ • $y = \cos t$ has a derivative $\frac{dy}{dt} = -\sin t$	By inspecting the graphs of cosine and sine, and the gradients of their tangents, students conjecture that $(\cos t)' = -\sin t$ and $(\sin t)' = \cos t$. Numerical estimations of the limits and informal proofs based on geometric constructions are used. The derivative of $\sin t$ or $\cos t$ from first principles is not required but provides an extension of the first principle process to calculate derivatives. A first-principles calculation from the limit definition provides a proof that the derivatives exist, and enables students to see the results in the context of the ‘gradient of tangent’ interpretation of derivatives from which the limit definition is derived. This traditional calculation proceeds by first computing the limit $$\lim_{\theta \to 0} \frac{\sin \theta}{\theta}$$ which is derived graphically, numerically, and geometrically. The derivatives of the sine and cosine functions in general can be derived from this special case using the Sums to Product identities.
How are the chain, product, and quotient rules applied with trigonometric functions? • The use of the quotient rule on $\frac{\sin t}{\cos t}$ allows the derivative of tan t to be found. • Derivatives can be found for functions such as $x \sin x$, $\frac{e^x}{\cos x}$. • The chain rule can be applied to $\sin f(x)$ and $\cos f(x)$.	This is an extension of differentiation rules, with a wider range of functions used to ‘build’ the composite functions. The reciprocal trigonometric functions: sec, cot and cosec, are not required in this subject.

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What types of problems can be solved by finding the derivatives of trigonometric functions? Derivatives can be used to: • find the slope of tangents to the graphs of trigonometric graphs • find the instantaneous velocity, when an object's displacement is described by a trigonometric function • determine the rate of change, the position of any local maxima or minima, and when the function is increasing or decreasing.	These applications of calculus have been developed earlier, using polynomials and linear combinations of power functions, and using exponential functions.
Use trigonometric functions and their derivatives to solve practical problems where trigonometric functions model actual situations.	Trigonometric functions are used to model many periodic scenarios, such as tidal heights, temperature changes, and AC voltages.

Sub-topic 1.5: The second derivative

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What is meant by 'the second derivative'? • The second derivative is the result of differentiating the derivative of a function. • The notation y'', $f''(x)$ and $\frac{d^2y}{dx^2}$ can be used for the second derivative.	Briefly consider the role of a derivative in describing the rate of change, to enable students to understand that the second derivative describes the rate of change of the first derivative. Graphical examples, where the rate of change is interpreted as the slope of the tangent, are used to show the relationships between the function, its derivative, and its second derivative.
What role does the second derivative play when studying motion along a straight line? • The second derivative of a displacement function describes the acceleration of a particle, and is used to see when the velocity is increasing or decreasing.	The use of acceleration functions to describe the changes in the velocity is a direct application of the second derivative's role in describing the rate of change of the first derivative.

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How can the first and second derivative of a function be used to locate stationary points and points of inflection? • Stationary points occur when the first derivative is equal to zero, and may be local maxima, local minima, or stationary inflections. • Points of inflection occur when the second derivative equals zero and changes sign, and may be classed as stationary or non-stationary. • The second derivative can be used to describe the concavity of a curve. • Whether the second derivative is positive, negative, or zero at a stationary point is used to determine the nature of the stationary point.	Graphical analysis of functions, their derivatives, and second derivatives (using electronic technology) are used to explore the rules about turning points and points of inflection. Interpreting sign diagrams of the first and second derivative provides information to assist in sketching the graphs of functions.

Topic 2: Discrete Random Variables

Subtopic 2.1: Discrete Random Variables

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What is a random variable? • a random variable is a variable, the value of which is determined by a process, the outcome of which is open to chance. For each random variable, once the probability for each value is determined it remains constant.	Random variables such as the number of heads appearing on a coin tossed 10 times, the number of attempts taken to pass a driving test and the length of a time before a phone loses its battery charge may be suitable to use to introduce the concept.
How are discrete random variables different from continuous random variables? • continuous random variables may take any value (often within set limits) • discrete random variables may only take specific values	Examples of continuous random variables include • height • mass. Examples of discrete random variables include the number of: • cars arriving at a set of traffic lights before it turns from red to green • phone calls made before a salesperson has sold 3 products • mutations on a strand of DNA • patients in a doctor's waiting room at any specific time • tropical cyclones annually, in a specific region.
What is a probability distribution of a discrete random variable and how can it be displayed? • a probability function specifies the probabilities for each possible value of a discrete random variable. This collection of probabilities is known as a probability distribution • a table or probability bar chart can show the different values and their associated probability • the sum of the probabilities must be 1.	A suitable context to explore is the profit obtained by a tradesperson who quotes $1000 for a particular job. Three outcomes that each have a probability of occurring are that the: • quote is rejected because it was too high, resulting in zero profit • job is done costing $800 in materials and labour, resulting in a $200 profit • job is done costing $1100 in materials and labour, resulting in a –$100 profit. It is appropriate to limit the scenarios to examples with variables that are a finite set of integers.
What is the difference between uniform discrete random variables and non-uniform discrete random variables? • For most discrete random variables the probabilities for the different outcomes are different, whereas uniform discrete random variables have the same probability for each outcome.	A comparison of the probabilities of rolling a single regular dice with the probabilities of rolling two dice simultaneously can emphasise the difference between uniform and non-uniform discrete random variables.

How can estimates of probabilities be obtained for discrete random variables from relative frequencies and probability bar charts?

- when a large number of independent trials is considered, the relative frequency of an event gives an approximation for the probability of that event

Frequency tables and probability bar charts for simple discrete random variables (such as the number of times each student must roll a dice before getting a 6) are converted into probability distributions, which are then interpreted.

Dice with 'alternative labelling' (such as 1,1,2,3,4,5 on the six sides) are also to create tables showing relative frequencies. Increasing the number of rolls of such dice shows how the approximation of the relative frequency approaches the theoretical probability. Electronic technology is used to simulate very large sets of data.

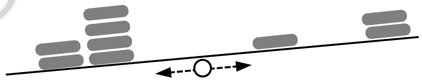
How can the expected value of a discrete random variable be calculated and used?

- the expected value of a discrete random variable is calculated using $E(X) = \sum_x x.p(x)$ where $p(x)$ is the probability function for achieving result x.
- the principal purpose of the expected value is to be a measurement of the centre of the distribution
- the expected value can be interpreted as a long-run sample mean.

Probability distributions for familiar scenarios, such as the number of burgers purchased by people who enter a takeaway outlet, are used to show the usefulness of the expected value.

The expected value for the previously described scenario of the tradesperson quoting $1000 for a job would give the tradesperson guidance on whether the quotes are too high or too low.

An analogy for the expected value is the balancing point: if the probability bar chart is considered as stacked weights on a movable pivot then the system balances when the pivot is placed at the position of the expected value.



It is appropriate at this point to introduce discrete random variables that do not have a real-world context, but are given in mathematical form such as:

x	2	5	8
$\Pr(X = x)$	0.40	0.25	0.35

How is the standard deviation of a discrete random variable calculated and used?

- the standard deviation of a discrete random variable is calculated $\sigma = \sqrt{\sum_x [x - E(X)]^2 p(x)}$ where $E(X)$ is the expected value and $p(x)$ is the probability function for achieving result x.
- the principal purpose of the standard deviation is to be a measurement of the spread of the distribution

The spreads of discrete random variables are compared using the standard deviation.

Subtopic 2.2: The Bernoulli Distribution

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What is a Bernoulli random variable, and where are they seen? • Discrete random variables with only two outcomes are called Bernoulli random variables. These two outcomes are often labelled ‘success’ and ‘failure’. • The Bernoulli distribution is the possible values and their probabilities of a Bernoulli random variable	The probability distribution of different Bernoulli random variables can be introduced by considering scenarios such as tossing a coin or rolling a dice that has different numbers of sides painted with one of two colours. These probabilities can be shown in a table or on a probability bar chart. One parameter, p, the probability of ‘success’, is used to describe Bernoulli distributions. The formula $p(x) = p^n(1 - p)^{1-n}$, for $n = 1$ signifying ‘success’ and $n = 0$ signifying ‘failure’ is used for Bernoulli distributions. Other examples of Bernoulli distributions include whether an email is read within an hour of being sent, and the random selection of a tool and discovering whether it is defective or not.
What is the mean and standard deviation of the Bernoulli distribution? • The mean of the Bernoulli distribution is p, and the standard deviation is given by $\sqrt{p(1-p)}$.	Investigation of the means of different Bernoulli distributions (calculated using $E(X) = \sum_x x.p(x)$) leads students to conjecture that the mean of a Bernoulli distribution is equal to p. This result can be proved using the algebraic formula for Bernoulli distributions. A similar investigation for the standard deviation may be used with guidance for formulating the conjecture, followed by completion of the algebraic proof. In this subject, the principal purpose of these values is to determine the mean and standard deviation for the binomial distribution.

Subtopic 2.3: Repeated Bernoulli Trials and the Binomial Distribution

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What is a binomial distribution and how is it related to Bernoulli trials? • When a Bernoulli trial is repeated, the number of successes is classed as a *Binomial random variable*. • The possible values for the different numbers of successes and their probabilities make up a *Binomial distribution.*	Students use tree diagrams to build simple Binomial distributions such as the number of heads resulting when a coin is tossed three times.
What is the mean and standard deviation of the binomial distribution? • The mean of the binomial distribution is *np*, and the standard deviation is given by $\sqrt{np(1-p)}$. Where *p* is the probability of success in a Bernoulli trial and *n* is the number of trials.	Both of these expressions can be built by considering a binomial variable as the sum of *n* Bernoulli variables. The mean and standard deviation of the Bernoulli distribution is used to obtain these expressions. In this subject, the principal purpose of these values is to determine the mean and standard deviation for the distribution of sample proportions in the 'Sampling and Confidence Intervals' topic.
When can a situation be modelled using the binomial distribution? • A binomial distribution is suitable when the number of trials is fixed in advance, the trials are independent, and each trial has the same probability of success.	Students classify scenarios as ones that can or cannot be modelled with a binomial distribution. In a context where sampling is done without replacement (e.g. an opinion poll), the distribution is not strictly binomial. When the population is large in comparison with the sample, the binomial distribution provides an excellent approximation of the probability of success.
How can binomial probabilities be calculated? • The probability of *k* successes from *n* trials is given by $Pr(X = k) = C^n_k \, p^k \, (1 - p)^{n-k}$ where *p* is the probability of success in the single Bernoulli trial.	The tree diagram used to determine the probability of obtaining a 6 on a dice twice when the dice is rolled three times, is considered to show that the calculation of binomial probabilities involves: • calculation of the number of ways that *k* successes can occur within *n* events (using the concepts in 'Counting' at Stage 1) • probability of 'success' to the power of the required number of successes • probability of 'failure' to the power of the required number of failures.

How can electronic technology be used to calculate binomial probabilities?

Although an understanding of the binomial probability formula is a requirement of this subject, calculations of binomial probabilities in problem solving should be done using electronic technology.

Students, given the probability of success, calculate probabilities such as:

- exactly k success out of n trials
- at least k success out of n trials
- between k_1 and k_2 success out of n trials

What happens to the binomial distribution as n gets larger and larger?

- The binomial distribution for large values of n has a symmetrical shape that many students will recognise.

Students will not have been formally introduced to the normal distribution yet, but it may be familiar to many.

Probability bar charts for binomial distributions of n = 10, n = 100, n = 1000 are built using a spreadsheet (for a specific value of p). Comparing these graphs shows the shape approaching a normal distribution. Discussion on adding a 'smooth curve' to emphasise the shape leads to the concept of continuous random variables.

Topic 3: Integral Calculus

Sub-topic 3.1: Anti-differentiation

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
Is there an operation which is the reverse of differentiation? • Finding a function whose derivative is the given function is called 'anti-differentiation'. • 'Anti-differentiation' is more commonly called 'integration' or 'finding the indefinite integral'.	Briefly consider simple derivatives. Posing a question about determining the original function, if its derivative is known, introduces integral calculus.
What is the indefinite integral $\int f(x)\mathrm{d}x$? • Any function $F(x)$ such that $F'(x)=f(x)$ is called the indefinite integral of $f(x)$ • The notation used for determining the indefinite integrals is: $\int f(x)\ dx$	The formal language and notation of integration is introduced before the different processes are explored in detail.
Are there families of curves that have the same derivatives? • All families of functions of the form $F(x) + c$ for any constant c have the same derivative. Hence, if $F(x)$ is an indefinite integral of $f(x)$ then so is $F(x)+c$ for any constant c	
Of what types of functions can the indefinite What are some functions that can be integrated? • By reversing the differentiation processes, the integrals of x^n (for $n \neq -1$), e^x, $\sin x$ and $\cos x$ can be determined. • Reversing the differentiation processes and consideration of the chain rule can be used the determine the integrals of $[f(x)]^n$ (for $n \neq$ 1), $e^{f(x)}$, $\sin f(x)$ and $\cos f(x)$ for linear functions $f(x)$. • $\int [f(x)+g(x)]dx$ = $\int f(x)dx+\int g(x)dx$	The processes for evaluating these integrals are developed by considering the relevant differentiation processes, and examining what must be done to reverse them.

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What is needed to be able to determine a specific constant of integration? • When the value of the indefinite integral is known for a specific value of the variable (often an initial condition) the constant of integration can be determined.	Determine f(x), given f′(x) and f(a) = b.

Sub-topic 3.2: The Area under Curves

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How can the area under a curve be estimated? • The area under a simple positive monotonic curve is approximated by upper and lower sums of the areas of rectangles of equal width	This sub-topic can be introduced by briefly considering the areas of polygons, then discussing the difficulty of generalising this concept to figures with curved boundaries. Students start with simple graphs where area is worked out using geometric principles (e.g. cross-sectional areas, and the determination of approximation of π or other irrational numbers).
How can the estimate of the area be improved? • Decreasing the width of the rectangles improves the estimate of the area, but makes it more cumbersome to calculate.	After students have calculated areas under a curve using electronic technology, they consider what process the technology is using to evaluate these areas by addition and multiplication operations only.

Key Questions and Key Concepts

How can integration be used to find the *exact* value of the area under a curve?

- The exact value of the area is the unique number between the upper and lower sums, which is obtained as the width of the rectangles approaches zero.
- The definite integral $\int_a^b f(x)\mathrm{d}x$ can be interpreted as the exact area of the region between the curve $y = f(x)$ and the x-axis over the interval $a \le x \le b$ (for a positive continuous functions $f(x)$.

How is the area above a function that is below the *x*-axis calculated?

- When $f(x)$ is a continuous negative function the exact area of the region between the curve $y = f(x)$ and the x-axis over the interval $a \le x \le b$ is given by $-\int_a^b f(x)\mathrm{d}x$

How is the area between the functions $f(x)$ and $g(x)$ over the interval $a \le x \le b$ calculated?

- When f(x) is above g(x), the area is given by $\int_a^b [f(x) - g(x)]dx$

Considerations for Developing Teaching and Learning Strategies

Students are introduced to the correct notation and use electronic technology to calculate areas. In many cases the technology will only give an approximation for the area, leading to the need for an algebraic process to calculate the *exact* value of definite integrals.

This sub-topic can be developed without reference to the algebraic computation of the antiderivative.

The computation of carefully chosen integrals (using electronic technology) leads students to observe the following results:

$$\int_a^a f(x)\mathrm{d}x = 0$$

$$\int_a^b f(x)\mathrm{d}x + \int_b^c f(x)\mathrm{d}x = \int_a^c f(x)\mathrm{d}x.$$

Sub-topic 3.3: Fundamental Theorem of Calculus

Key Questions and Key Concepts

How can exact values of definite integrals be calculated?

- The statement of the fundamental theorem of calculus

$$\int_a^b f(x)\mathrm{d}x = F(b) - F(a),$$

where $F(x)$ is such that $F'(x) = f(x)$.

Considerations for Developing Teaching and Learning Strategies

The fundamental theorem of calculus is now introduced to address the need for exact values for areas under curves.

What results can be interpreted from the direct application of the fundamental theorem of calculus?

- $\int_a^a f(x)\mathrm{d}x = 0$
- $\int_a^b f(x)\mathrm{d}x + \int_b^c f(x)\mathrm{d}x = \int_a^c f(x)\mathrm{d}x.$

These two results may have been 'discovered' when the areas under curves were being explored. They can now be verified by applying the fundamental theorem of calculus.

The theorem $F'(x) = \dfrac{d}{dx}\int_a^x f(t)\ dt = f(x)$ can be explored, and proved geometrically to show that the fundamental theorem of calculus links the processes of differentiation and integration.

How can the fundamental theorem of calculus be applied to evaluate areas?

- In the exploration of areas in sub-topic 3.2, the use of technology meant that the exact value of areas (or the exact value of one of the end points of the area) could not always be obtained. The fundamental theorem of calculus can be used in those circumstances.

Sub-topic 3.4: Applications of integration

Key Questions and Key Concepts

What useful information can be obtained by calculating the area under a curve?

- Applications can be modelled by functions, and evaluating the area under or between curves can be used to solve problems.
- When the rate of change of a quantity is graphed against the elapsed time, the area under the curve is the total change in the quantity.

How can integration be used in motion problems?

- The total distance travelled by an object is determined from its velocity function.
- An object's position is determined from its velocity function if the initial position (or position at some specific time) is known.
- An object's velocity is determined from its acceleration function if the initial velocity (or velocity at some specific time) is known.

Considerations for Developing Teaching and Learning Strategies

The types of functions that may be suitable for this sub-topic are given in sub-topic 3.1.

When curves describe, for example, caves, tunnels, and drainage pipe cross-sections, the calculation of different areas has practical applications.

The following examples of rates of change models can be considered:

- the rate at which people enter a sports venue
- water flow during a storm
- the velocity of a vehicle
- traffic flow through a city
- the electricity consumption of a household.

Topic 4: Logarithmic Functions

Sub-topic 4.1: Using logarithms for solving exponential equations

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How can an exact solution to an equation where the power is the unknown quantity be obtained? • The solution for *x* of the exponential equation $a^x = b$ is given using logarithms $x = \log_a b$	Logarithms were introduced in sub-topic 5.3 in the Stage 1 topic, 'Growth and Decay'. Logarithms are introduced here for their use in algebraically solving exponential equations. Briefly consider the definition of a logarithm as a number, and the rules for operating with logarithms.
Given $y = e^x$, what value of x will produce a given value of y? • The natural logarithm is the logarithm of base *e*. • When $y = e^x$ then $x = \log_e y = \ln y$ • Natural logarithms obey the laws: $\ln a^b = b\ln a$ $\ln ab = \ln a + \ln b$ $\ln \dfrac{a}{b} = \ln a - \ln b$	Examining a table of the non-negative integer powers of e enables students to understand the extent and rapidity of exponential growth, and also raises the following questions: • In a population growth model, $P = P_0 e^{kt}$, when will the population reach a particular value? • In a purely mathematical sense, what power of e gives 2, or 5, and so on? Looking for some of these values (by trial and error with a calculator) gives approximations for some specific natural logarithms, encountered previously as the multipliers for the derivatives of 2^x and 5^x.
How are natural logarithms used to find the solutions for problems involving applications of differential calculus with exponential functions? Derivatives are used to: • find the slope of tangents to the graphs of functions • find the instantaneous velocity, when an object's displacement is described by an exponential function • determine the rate of change, the position of any local maxima or minima, and when the function is increasing or decreasing.	This subtopic allows for extension of the problem-solving approaches used in exploring applications of differential calculus. The use of logarithms allows solutions to be found. An example of suitable functions to explore is the logistic function: $P(t) = \dfrac{L}{1 + Ae^{-bt}}$

Sub-topic 4.2: Logarithmic Functions and their Graphs

Key Questions and Key Concepts

Where and why are logarithmic scales used?

- In many areas of measurement a logarithmic scale is used to render an exponential scale linear or because the numbers cover too large a range to make them easy to use

What are the features and shape of the graph of $y = \ln x$ and its translations?

- The graph of $y = \ln x$ is continuously increasing, with a x-intercept at x = 1 and a vertical asymptote x = 0.
- The graph of $y = k \ln (bx + c)$ is the same shape as the graph of $y = \ln x$, with the values of k, b, and c determining its specific characteristics.

How are the graphs of $y = e^x$ and $y = \ln x$ related?

- Like all inverse functions, the graphs of $y = e^x$ and $y = \ln x$ are reflections of each other in the line $y = x$

Considerations for Developing Teaching and Learning Strategies

Examples of suitable logarithmic scales include magnitude of earthquakes (Richter scale), loudness (decibel scale), acidity (pH scale), brightness of stars (Pogson's scale of apparent magnitude).

The study of these scales is used to reinforce the nature of the logarithmic function.

Electronic technology can be used to graph $y = \log_a x$ for different values of *a*, which enables students to formulate conjectures about the properties of this family of curves.

Similarly, electronic technology can be used to explore the translations (caused by k, b and c) within the graph of $y = k \ln (bx + c)$

Sub-topic 3.3: Calculus of Logarithmic Functions

What are the derivatives of $y = \ln x$ and $y = \ln f(x)$?

- The function $y = \ln x$ has a derivative

$$\frac{dy}{dx} = \frac{1}{x}$$

- The function $y = \ln f(x)$ has a derivative

$$\frac{dy}{dx} = \frac{f'(x)}{f(x)}.$$

What is the indefinite integral of $\frac{1}{x}$?

- Provided x is positive, $\int \frac{1}{x} dx = \ln x + c$

Using electronic technology to study the slope of tangents along the curve $y = \ln x$ leads students to the conjecture that $\frac{dy}{dx} = \frac{1}{x}$. The proof (using implicit differentiation) is not required in this subject.

The derivative of $y = \ln f(x)$ can be established using the chain rule.

The integral of $\frac{1}{x}$ can be considered by reviewing the relationships between functions and their derivatives through anti-differentiation.

What types of problems can be solved by finding the derivatives of logarithmic functions?

The derivative of a logarithmic functions can be used to:

- find the slope of tangents to the function
- find the instantaneous velocity, when an object's displacement is described by a logarithmic function
- determine the rate of change, the position of any local maxima or minima, and when the function is increasing or decreasing

Use logarithmic functions and their derivatives to solve practical problems.

Logarithmic functions can be used to model many scenarios, such as the magnitude of earthquakes, loudness, and acidity.

Topic 5: Continuous Random Variables and the Normal Distribution

Subtopic 5.1: Continuous Random Variables

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What is a continuous random variable? • A continuous random variable can take any value (sometimes within set limits)	A comparison between discrete and continuous random variables can be made using people's ages and heights.
How can the probabilities associated with continuous random variables be estimated? • The probability of each specific value of a continuous random variable is effectively zero. The probabilities associated with a specific range of values for a continuous random variable can be estimated from relative frequencies and from histograms.	An exploration of histograms for decreasing width ranges of continuous random variables shows that a smooth curve would be suitable to illustrate probabilities.
How are probability density functions used? • A probability density function is a function that describes the relative likelihood for the continuous random variable to be a given value. • A function is only suitable to be a probability density function if it is continuous and positive over the domain of the variable. Additionally, the area bound by the curve of the density function and the x-axis must equal 1, when calculated over the domain of the variable. • The area under the probability density function from a to b gives the probability that the values of the continuous random variable are between a and b.	The properties of all probability density functions: can be explored using: the uniform function *f(x)* = $\frac{1}{10}$ for $0 \le x \le 10$, the linear function *f(x)* = $\frac{x}{4}$ for $1 \le x \le 3$, the quadratic function *f(x)* = $\frac{3x}{2} - \frac{3x^2}{4}$ for $0 \le x \le 2$ These functions would give the opportunity to revise the evaluation of areas using definite integrals (calculated using the fundamental theorem of calculus or using electronic technology). The fundamental theorem of calculus can be used for questions such as: determine the value of *k* that makes $f(x) = ke^{-x}$ a probability density function in the domain $0 \le x \le 1$
How are the mean and standard deviation of continuous random variables calculated? • the mean is calculated using *E(x)* = $\int_{-\infty}^{\infty} x\, f(x)\, dx$ • the standard deviation is calculated using $\sigma = \sqrt{\int_{-\infty}^{\infty} [x - E(x)]^2 f(x)\, dx}$	This section reinforces understanding of the usefulness of the mean as the expected value of a random variable – the long-run sample mean. Similarly, this section reinforces the role of the standard deviation in describing the spread of a random variable.

Subtopic 5.2: Normal Distributions

Key Questions and Key Concepts

What are normal random variables, and for what scenarios do they provide a suitable model?

- Natural variation means that many random variables have the majority of values close to the mean, with symmetrically lower probabilities as the values get further above or below the mean. Under some conditions these variables can be classed as *normal random variables.*

What are the key properties of normal distributions?

- The normal distribution is symmetric and bell-shaped. Each normal distribution is determined by the mean μ and the standard deviation σ.

What is the probability density function for the normal distribution?

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}.$$

How can the percentage of a population meeting a certain criterion be calculated for normal distributions?

- When the normal distribution is used to model data, the area under the curve within an interval is interpreted as a proportion or a probability.

How can the normal distribution be used to determine the value above or below which a certain proportion lies?

- When the other limit is known, the upper or lower limit of a known area can be calculated.

What is the standard normal distribution?

- All normal distributions can be transformed to the standard normal distribution with $\mu = 0$ and $\sigma = 1$ by using the formula:

$$Z = \frac{X - \mu}{\sigma}$$

Considerations for Developing Teaching and Learning Strategies

When investigating an explanation for why normal distributions occur, students build a spreadsheet that generates a large amount of data from the sum of a finite number of random numbers. Potentially useful examples are found in manufacturing processes (e.g. the length of a concrete sleeper or the weight of a container of butter).

Investigations of different normal distributions demonstrate the symmetrical shape, the position of the mean, and the standard deviation's role in determining the spread.

The importance of the standard deviation is emphasised using the '68 : 95: 99.7%' rule.

With the aid of electronic technology, students explore the features of the graph of the function

$f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$ to confirm that it is a probability density function. Different values of μ and σ are used to explore the effects of these parameters on transforming the density function for different distributions.

The position of the inflections, in terms of standard deviations, is explored using the skills learned in the Calculus topics.

The normal distribution functions of electronic technology should be used, rather than evaluating areas using the probability density function for calculations of proportions or probabilities.

Electronic technology should be used to calculate the upper or lower bounds of an interval within a normal distribution that contain a specific proportion of the population.

Through calculations, students develop an understanding that, for all normal curves, the value of $Pr(\mu - a\sigma \le X \le \mu + b\sigma)$ does not depend on μ and σ, and is equal to $Pr(a \le Z \le b)$.

Z can be interpreted as the number of standard deviations by which X lies above or below the mean.

The standard normal distribution provides one basis for the derivation of confidence intervals in the 'Sampling and Confidence Intervals' topic.

Subtopic 5.3: The Central Limit Theorem

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies

How can the outcome of adding of *n* independent observations of X be described?

Let X be a random variable for which $\mathrm{E}(X) = \mu$ and $\mathrm{SD}(X) = \sigma$.

- For *n* independent observations of X,
  $$X_1, X_2, X_3, \ldots, X_n,$$
  the sum of the observations is the random variable
  $$S_n = X_1 + X_2 + X_3 + \ldots + X_n,$$

What are $\mathrm{E}(S_n)$ and $\mathrm{SD}(S_n)$?

$$\mathrm{E}(S_n) = n\mu \text{ and } \mathrm{SD}(S_n) = \sigma\sqrt{n}$$

- The distribution of S_n depends on the distribution of X and the magnitude of *n*.

How can the outcome of averaging *n* independent observations of X be described?

- The mean of the observations is the random variable
  $$\bar{X}_n = \frac{X_1 + X_2 + X_3 + \ldots + X_n}{n},$$

where

$$\mathrm{E}(\bar{X}_n) = \mu \text{ and } \mathrm{SD}(\bar{X}_n) = \frac{\sigma}{\sqrt{n}}.$$

- The distribution of $\bar{X}_n$ depends on the distribution of X and the magnitude of *n*.

How are S_n and $\bar{X}_n$ related?

$$\bar{X}_n = \frac{S_n}{n}$$

What are the distributions S_n and $\bar{X}_n$ called?

- The distributions S_n and $\bar{X}_n$ are called *sampling distributions*. Because of the relationship $\bar{X}_n = \frac{S_n}{n}$, the two distributions have the same form.

What is the form of the sampling distributions S_n and $\bar{X}_n$ if the distribution of X normally distributed?

The outcome of one roll of a fair dice can be described by the discrete random variable X, which has a uniform distribution of values {1, 2, 3, 4, 5, 6}.

If a fair dice is rolled *n* times such that each roll is independent of any other roll, the sequence of outcomes

$$X_1, X_2, X_3, \ldots, X_n$$

is a sequence of *n* independent observations from X.

The sum of *n* independent rolls (a random sample of rolls) of a fair dice,
$S_n = X_1 + X_2 + X_3 + \ldots + X_n$, is a random variable.

The mean value of *n* independent rolls of a fair dice, $\bar{X}_n = \dfrac{X_1 + X_2 + X_3 + \ldots + X_n}{n}$ is a random variable.

Experiment with dice (real and/or digital) to determine estimates for the form of the distributions of S_n and $\bar{X}_n$ as well as the mean and standard deviation of the distributions for various values of *n*.

When *n* individuals are sampled randomly (with replacement) from a population, and the value of a certain variable X is recorded for each sampled individual, the sequence of outcomes,

$$X_1, X_2, X_3, \ldots, X_n$$

is a sequence of *n* independent observations from X.

The sum of *n* randomly sampled values of X,
$S_n = X_1 + X_2 + X_3 + \ldots + X_n$, is a random variable.

The mean value of *n* randomly sampled values of X, $\bar{X}_n = \dfrac{X_1 + X_2 + X_3 + \ldots + X_n}{n}$ is a random variable.

Use digital sampling from finite populations of different forms to determine estimates for the form of the distributions of S_n and $\bar{X}_n$ as well as the mean and standard deviation of the distributions for various values of *n*.

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
• For *n* independent observations of X, $X_1, X_2, X_3, \ldots, X_n$, where $X \sim N(\mu, \sigma)$, $S_n \sim N(n\mu, \sigma\sqrt{n})$ and $\bar{X}_n \sim N(\mu, \frac{\sigma}{\sqrt{n}})$	
These are results of random variable theory.	
What is the form of the sampling distributions S_n and $\bar{X}_n$ if the distribution of X non-normally distributed? • If X is a ***non-normally*** distributed random variable with $\mathrm{E}(X) = \mu$ and $\mathrm{SD}(X) = \sigma$, then for *n* independent observations of X, $X_1, X_2, X_3, \ldots, X_n$, S_n is approximately $N(n\mu, \sigma\sqrt{n})$ and $\bar{X}_n$ is approximately $\bar{X}_n \sim N(\mu, \frac{\sigma}{\sqrt{n}})$ provided *n* is sufficiently large. This is a consequence of the Central Limit Theorem. Note: It is not a formal statement of the Central Limit Theorem.	Experiment with a variety of different shaped population distributions and various values of *n* to appreciate that the magnitude of *n* that results in approximate normality of the sampling distribution depends on the degree of non-normality of the population distribution.
What is a *simple random sample* of observations from a population? • A simple random sample of size *n* is a group of *n* subjects chosen from a population in such a way that every possible sample of size *n* has an equal chance of being of selected.	A simple random sample can be constructed by choosing the first individual so that all N members of the population are equally likely to be chosen. The second individual is chosen so that all of the remaining N-1 members of the population are equally likely. The process is repeated until n individuals are chosen.
How is a simple random sample linked to random variable theory? • If one *simple random sample* of *n* individuals is chosen from a population, and the value of a certain variable X is recorded for each individual in the sample, the mean of the *n* values for this one sample is denoted $\bar{x}$. • If $\mathrm{E}(X) = \mu$ and $\mathrm{SD}(X) = \sigma$ then an approximation for the probability that $\bar{X}$ lies in a certain range be calculated using the consequence of the Central Limit Theorem that, $$Z_n = \frac{\bar{X}_n - \mu}{\frac{\sigma}{\sqrt{n}}}$$ where $Z_n \sim N(0,1)$.	The value of X for a single individual chosen randomly from the population is a random variable, the distribution of which can be represented by the relative frequency distribution or the relative frequency histogram for the population. If the sample size n is small relative to the population size N the values $X_1, X_2, X_3, \ldots, X_n$ behave approximately like independent random variables from that distribution.

How can an approximation for the probability that $\bar{x}$ lies in a certain range be calculated if $\mathrm{SD}(X)$ is unknown?

- If one *simple random sample* of *n* individuals is chosen from a population and the value of a certain variable X is recorded for each individual in the sample, the standard deviation of the *n* values is denoted *s*.

If $\mathrm{E}(X)=\mu$ then

$$Z_n \approx \frac{\bar{X}_n-\mu}{\frac{s}{\sqrt{n}}}$$

where $Z_n \sim N(0,1)$.

Electronic sampling can be used to illustrate this result.

Topic 6: Sampling and Confidence Intervals

Subtopic 6.1: Confidence Intervals for a Population Mean

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What type of random variable is a sample mean? • If a sample is chosen and its mean calculated then the value of that sample mean will be variable. Different samples will yield different sample means. • Sample means are continuous random variables. • A practical consequence of the central limit theorem is that, for sufficiently large sample size, the distribution of sample means will be approximately normal. • The distribution of sample means will have a mean equal to μ, the population mean. This distribution has a standard deviation equal to $\frac{\sigma}{n}$, where σ is the standard deviation of the population and *n* is the sample size.	Students are familiar with the concept of sample means from study of the Central Limit Theorem. Briefly consider that a sample mean cannot be used to make definitive statements about the population mean, but can be used as a guide only. Students explored the distribution of sample means when learning about the Central Limit Theorem.
How can a single sample mean (of appropriately large sample size) be used to create an interval estimate for the population mean? • An interval can be created around the sample mean that will be expected, with some specific confidence level, to contain the population mean.	Knowing the approximately normal distribution of sample means and the '68 : 95: 99.7%' rule, it can be seen that approximately 68% of sample means will be within one standard deviation of the unknown population mean and that approximately 95% of sample means will be within two standard deviations of the unknown population mean.
How can the upper and lower limits of a confidence interval for the population mean be calculated? • If $\bar{x}$ is the sample mean and *s* the standard deviation of the sample, then the interval: $\bar{x} - z\frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + z\frac{s}{\sqrt{n}}$ can be created. The value of *z* is determined by the confidence level required.	This formula can be introduced by considering the approximate 68% confidence interval using *z* = 1 and the approximate 95% confidence interval using *z* = 2. The creation of both of these intervals shows students that σ should be used if available and not *s*, but rarely is σ known when μ is not. Other confidence levels (say 90% and 98%) require the calculation of *z* using students' knowledge of the standard normal distribution and of how to calculate (using electronic technology) the upper or lower bounds of an interval within a normal distribution that contains a specific proportion of the population. Although an understanding of the confidence interval formula is a requirement of this subject, calculations of confidence intervals in problem solving should be done using electronic technology.
Can a confidence interval be used to state facts about a population mean? • Not all confidence intervals will contain the true population mean. • The inclusion or not of a claimed population mean within a confidence interval can be used a guide to whether the claim is true or false, but definitive statements are not possible.	Simulation (using electronic technology to calculate the different intervals) can show the variation in confidence intervals (especially their approximate 'margin of error': $z\frac{s}{\sqrt{n}}$) due to different sample means, different sample standard deviations, and different confidence levels.

Subtopic 6.2: Population Proportions

Key Questions and Key Concepts

What is a population proportion and what does it represent?

- A population proportion p is the proportion of elements in a population that have a given characteristic. *p* is usually given as a decimal or fraction.
- A population proportion represents the probability that one element of the population, chosen at random, has the given characteristic being studied.

What is a sample proportion?

- If a sample of size n is chosen, and *X* is the number of elements with a given characteristic, then the sample proportion $\hat{p}$ is equal to $\frac{X}{n}$.

What type of variable is a sample proportion?

- A sample proportion is a discrete random variable. The distribution has a mean of *p* and a standard deviation of $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.

What happens to the distribution of $\hat{p}$ for large samples?

- As the sample size increases, the distribution of $\hat{p}$ becomes more and more like a normal distribution.

Considerations for Developing Teaching and Learning Strategies

Discussions about the number of left-handed students in a class can introduce the concept of a sample proportion. Compare this to the proportion of a population that is left-handed. By quickly surveying nearby classes, the effect of sample size on how close a sample proportion is to a known population proportion can be explored.

To justify that a sample proportion is discrete students consider a sample size of 10. Sample proportions of $\frac{5}{10}$ (0.5) and $\frac{6}{10}$ (0.6) would be possible, but a sample proportion of 0.55 would not.

The mean and standard deviation can easily be derived from the mean and standard deviation of the Binomial distribution.

Subtopic 6.3: Confidence Intervals for a Population Proportion

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
Can facts about a population proportion be gained from a sample proportion? • An interval can be created around the sample proportion that will be expected, with some specific confidence level, to contain the population proportion.	
How can the upper and lower limits of a confidence interval for the population proportion be calculated? • If $\hat{p}$ is the sample proportion, then the interval: $\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ can be created. The value of z is determined by the confidence level required.	This formula matches the structure of the formula for the confidence interval for a population mean, and allows for revision of its creation by considering the position within the distributions (compared to the population proportion) of individual samples proportions. Although an understanding of the confidence interval formula is a requirement of this subject, calculations of confidence intervals in problem solving should be done using electronic technology.
Can a confidence interval be used to state facts about a population proportion? • Not all confidence intervals will contain the true population proportion. • The inclusion or not of a claimed population proportion within a confidence interval can be used a guide to whether the claim is true or false, but definitive statements are not possible.	Simulation (using electronic technology to calculate the different intervals) can show the variation in confidence intervals (especially their approximate 'margin of error': $z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$) due to different sample proportions and different confidence levels.

ASSESSMENT SCOPE AND REQUIREMENTS

All Stage 2 subjects have a school assessment component and an external assessment component.

EVIDENCE OF LEARNING

The following assessment types enable students to demonstrate their learning in Stage 2 Mathematical Methods.

School Assessment (70%)

- Assessment Type 1: Skills and Applications Tasks (50%)
- Assessment Type 2: Mathematical Investigation (20%)

External Assessment (30%)

- Assessment Type 3: Examination (30%)

Students provide evidence of their learning through eight assessments, including the external assessment component. Students complete:

- five skills and applications tasks
- two mathematical investigations
- one examination.

It is anticipated that from 2018 all school assessments will be submitted electronically.

ASSESSMENT DESIGN CRITERIA

The assessment design criteria are based on the learning requirements and are used by:

- teachers to clarify for the student what he or she needs to learn
- teachers and assessors to design opportunities for the student to provide evidence of his or her learning at the highest possible level of achievement.

The assessment design criteria consist of specific features that:

- students should demonstrate in their learning
- teachers and assessors look for as evidence that students have met the learning requirements.

For this subject, the assessment design criteria are:

- concepts and techniques
- reasoning and communication.

The specific features of these criteria are described below.

The set of assessments, as a whole, must give students opportunities to demonstrate each of the specific features by the completion of study of the subject.

Concepts and Techniques

The specific features are as follows:

CT1 Knowledge and understanding of concepts and relationships

CT2 Selection and application of techniques and algorithms to solve problems in a variety of contexts

CT3 Development and application of mathematical models

CT4 Use of electronic technology to find solutions to mathematical problems.

Reasoning and Communication

The specific features are as follows:

RC1 Interpretation and evaluation of mathematical results, with an understanding of their reasonableness and limitations

RC2 Knowledge and use of appropriate mathematical notation, representations, and terminology

RC3 Communication of mathematical ideas and reasoning, to develop logical arguments

RC4 Development, testing, and proof of valid conjectures.

SCHOOL ASSESSMENT

Assessment Type 1: Skills and Applications Tasks (50%)

Students complete five skills and applications tasks.

Skills and applications tasks are completed under the direct supervision of the teacher.

Students find solutions to mathematical problems that may:
- be routine, analytical, and/or interpretative
- be posed in a variety of familiar and new contexts
- require discerning use of electronic technology.

In setting skills and applications tasks, teachers may provide students with information in written form or in the form of numerical data, diagrams, tables, or graphs. A task should require students to demonstrate an understanding of relevant mathematical concepts and relationships.

Students select appropriate techniques or algorithms and relevant mathematical information to find solutions to routine, analytical, and/or interpretative problems. Some of these problems should be set in context, for example, social, scientific, economic or historical.

Students provide explanations and arguments, and use correct mathematical notation, terminology, and representation throughout the task. They may be required to use electronic technology appropriately to aid and enhance the solution of some problems.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:
- concepts and techniques
- reasoning and communication.

Assessment Type 2: Mathematical Investigation (20%)

Students complete two mathematical investigations.

Students investigate mathematical relationships, concepts, or problems, which may be set in an applied context. The subject of a mathematical investigation may be derived from one or more subtopics, although it can also relate to a whole topic or across topics.

A mathematical investigation may be initiated by a student, a group of students, or the teacher. Teachers may give students a clear, detailed, and sequential set of instructions for part of the investigation or to initiate the investigation, or may provide guidelines for students to develop themes or aspects of their own choice. Teachers should give some direction about the appropriateness of each student's choice and guide and support students' progress in an investigation.

Teachers may need to provide support and clear directions for the first mathematical investigation. However, subsequent investigations should be less directed and set within more open-ended contexts.

Students demonstrate their problem-solving strategies as well as their knowledge, skills, and understanding in the investigation. They are encouraged to use a variety of mathematical and other software (e.g. CAS, spreadsheets, statistical packages) to enhance their investigation. The generation of data and the exploration of patterns and structures, or changing parameters, may provide an important focus. From these, students may recognise different patterns or structures. Notation, terminology, forms of representation of information gathered or produced, calculations, evidence of technological skills, and results are important considerations.

Students complete a report for each mathematical investigation.

In the report, students formulate and test conjectures, interpret and justify results, and draw conclusions. They give appropriate explanations and arguments.

The report may take a variety of forms, but would usually include the following:

- an outline of the problem and context
- the method required to find a solution, in terms of the mathematical model or strategy used
- the application of the mathematical model or strategy, including
  - relevant data and/or information
  - mathematical calculations and results, using appropriate representations
  - the analysis and interpretation of results, including consideration of the reasonableness and limitations of the results
- the results and conclusions in the context of the problem
- a bibliography and appendices, as appropriate.

The format of an investigation report may be written or multimodal.

Each investigation report should be a maximum of 15 pages if written, or the equivalent in multimodal form.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:

- concepts and techniques
- reasoning and communication.

EXTERNAL ASSESSMENT

Assessment Type 3: Examination (30%)

Students undertake a 3-hour external examination.

The examination is based on the ‘key questions and key concepts’ in the six topics. The ‘considerations for developing teaching and learning strategies’ are provided as a guide only, although applications described under this heading may provide contexts for examination questions.

The examination consists of a range of problems, some focusing on knowledge and routine skills and applications, and others focusing on analysis and interpretation. Some problems may require students to interrelate their knowledge, skills, and understanding from more than one topic. Students provide explanations and arguments, and use correct mathematical notation, terminology, and representation throughout the examination.

The examination is divided into two parts:

- Part 1 (40%, 60 minutes): Calculations without electronic technology (graphics/scientific calculators).
- Part 2 (60%, 120 minutes): Calculations with access to approved electronic technology (graphics/scientific calculators).

Students have 10 minutes in which to read both Part 1 and Part 2 of the examination. At the end of the reading time, students begin their answers to Part 1. For this part, students do not have access to electronic technology (graphics or scientific calculators).

At the end of the specified time for Part 1, students stop writing. Students submit Part 1 to the invigilator.

Students have access to Board-approved calculators for Part 2 of the examination. The invigilator coordinates the distribution of calculators (graphics and scientific). Once all students have received their calculators, the time allocated for Part 2 begins, and students resume writing their answers.

The SACE Board will provide a list of approved graphics calculators for use in Assessment Type 3: Examination that meet the following criteria:

- have flash memory that does not exceed 5.0 MB (this is the memory that can be used to store add-in programs and other data)
- can calculate derivative and integral values numerically
- can calculate probabilities
- can calculate with matrices
- can draw a graph of a function and calculate the coordinates of critical points using numerical methods
- solve equations using numerical methods
- do not have a CAS (Computer Algebra System)
- do not have SD card facility (or similar external memory facility).

Graphics calculators that currently meet these criteria, and are approved for 2017, are as follows:
Casio fx-9860G AU
Casio fx-9860G AU Plus
Hewlett Packard HP 39GS
Sharp EL-9900
Texas Instruments TI-83 Plus
Texas Instruments TI-84 Plus
Texas Instruments – TI 84 Plus C –silver edition
Texas Instruments – TI 84 Plus CE.

Other graphic calculators will be added to the approved calculator list as they become available.

Students may bring two graphics calculators or one scientific calculator and one graphics calculator into the examination room.

There is no list of Board-approved scientific calculators. Any scientific calculator, except those with an external memory source, may be used.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:

- concepts and techniques
- reasoning and communication.

PERFORMANCE STANDARDS

The performance standards describe five levels of achievement, A to E.

Each level of achievement describes the knowledge, skills, and understanding that teachers and assessors refer to in deciding how well a student has demonstrated his or her learning, on the basis of the evidence provided.

During the teaching and learning program the teacher gives students feedback on their learning, with reference to the performance standards.

At the student's completion of study of each school assessment type, the teacher makes a decision about the quality of the student's learning by:

- referring to the performance standards
- assigning a grade between A+ and E- for the assessment type.

The student's school assessment and external assessment are combined for a final result, which is reported as a grade between A+ and E-.

Performance Standards for Stage 2 Mathematical Methods

	Concepts and Techniques	Reasoning and Communication
A	Comprehensive knowledge and understanding of concepts and relationships. Highly effective selection and application of techniques and algorithms to find efficient and accurate solutions to routine and complex problems in a variety of contexts. Successful development and application of mathematical models to find concise and accurate solutions. Appropriate and effective use of electronic technology to find accurate solutions to routine and complex problems.	Comprehensive interpretation and critical evaluation of mathematical results, with an in-depth understanding of their reasonableness and possible limitations. Proficient and accurate use of appropriate mathematical notation, representations, and terminology. Highly effective communication of mathematical ideas and reasoning to develop logical and concise arguments. Effective development and testing of valid conjectures, with proof.
B	Some depth of knowledge and understanding of concepts and relationships. Mostly effective selection and use of mathematical techniques and algorithms to find accurate solutions to routine and some complex problems in a variety of contexts. Mostly successful development and application of mathematical models to find accurate solutions. Mostly appropriate and effective use of electronic technology to find accurate solutions to routine and some complex problems.	Mostly appropriate interpretation and some critical evaluation of mathematical results, with some depth of understanding of their reasonableness and possible limitations. Mostly accurate use of appropriate mathematical notation, representations, and terminology. Mostly effective communication of mathematical ideas and reasoning to develop logical arguments. Mostly effective development and testing of valid conjectures, with substantial attempt at proof.
C	Generally competent knowledge and understanding of concepts and relationships. Generally effective selection and use of mathematical techniques and algorithms to find mostly accurate solutions to routine problems in a variety of contexts. Some development and successful application of mathematical models to find generally accurate solutions. Generally appropriate and effective use of electronic technology to find mostly accurate solutions to routine problems.	Generally appropriate interpretation and evaluation of mathematical results, with some understanding of their reasonableness and possible limitations. Generally appropriate use of mathematical notation, representations, and terminology, with some inaccuracies. Generally effective communication of mathematical ideas and reasoning to develop some logical arguments. Development and testing of generally valid conjectures, with some attempt at proof.
D	Basic knowledge and some understanding of concepts and relationships. Some effective selection and use of mathematical techniques and algorithms to find some accurate solutions to routine problems in some contexts. Some application of mathematical models to find some accurate or partially accurate solutions. Some appropriate use of electronic technology to find some accurate solutions to routine problems.	Some interpretation and attempted evaluation of mathematical results, with some awareness of their reasonableness and possible limitations. Some appropriate use of mathematical notation, representations, and terminology, with some accuracy. Some communication of mathematical ideas, with attempted reasoning and/or arguments. Attempted development or testing of a reasonable conjecture.
E	Limited knowledge or understanding of concepts and relationships. Attempted selection and limited use of mathematical techniques or algorithms, with limited accuracy in solving routine problems. Attempted application of mathematical models, with limited accuracy. Attempted use of electronic technology, with limited accuracy in solving routine problems.	Limited interpretation or evaluation of mathematical results, with limited awareness of their reasonableness and possible limitations. Limited use of appropriate mathematical notation, representations, or terminology, with limited accuracy. Attempted communication of mathematical ideas, with limited reasoning. Limited attempt to develop or test a conjecture.

ASSESSMENT INTEGRITY

The SACE Assuring Assessment Integrity Policy outlines the principles and processes that teachers and assessors follow to assure the integrity of student assessments. This policy is available on the SACE website (www.sace.sa.edu.au) as part of the SACE Policy Framework.

The SACE Board uses a range of quality assurance processes so that the grades awarded for student achievement, in both the school assessment and the external assessment, are applied consistently and fairly against the performance standards for a subject, and are comparable across all schools.

Information and guidelines on quality assurance in assessment at Stage 2 are available on the SACE website (www.sace.sa.gov.au)

SUPPORT MATERIALS

SUBJECT-SPECIFIC ADVICE

Online support materials are provided for each subject and updated regularly on the SACE website (www.sace.sa.edu.au). Examples of support materials are sample learning and assessment plans, annotated assessment tasks, annotated student responses, and recommended resource materials.

ADVICE ON ETHICAL STUDY AND RESEARCH

Advice for students and teachers on ethical study and research practices is available in the guidelines on the ethical conduct of research in the SACE on the SACE website (www.sace.sa.edu.au).