YEAR 11 MATHEMATICAL METHODS **60 minutes**

FUNCTIONS AND GRAPHS TEST #1 **Calculator and A5 page of notes allowed.**

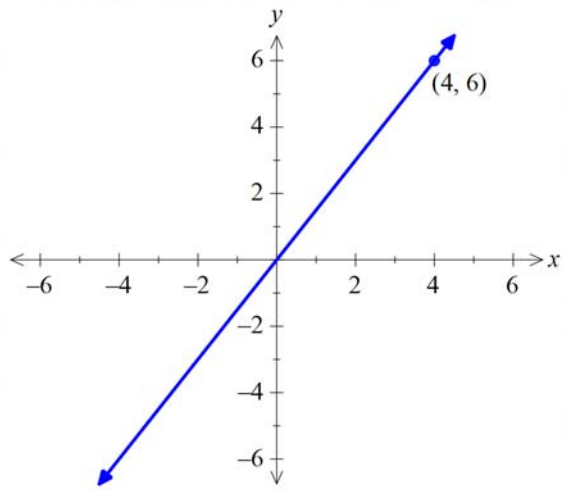
NAME: **52 Marks**

QUESTION 1 **4 marks**

(a) Consider the following graph.

(i) Explain why the variables are directly proportional.

(ii) Determine the equation connecting the variables



(2 marks)

(b) Consider the following table.

(i) State whether the variables are directly or inversely proportional

(ii) Determine the equation connecting the variables

a	-2	-1	1	2
y	3	6	-6	-3

(2 marks)

QUESTION 2 **8 marks**

If $f(x) = 5 - 2x$ and $g(x) = (x-1)^2$, find in simplest form

(a) $f(-1)$

(1 mark)

(b) $g(f(3))$

(2 marks)

(c) $f(3-x)$

(2 marks)

(d) Solve for x : $f(f(x)) = 3$

(3 marks)

QUESTION 3 **8 marks**

Consider the function $y = \dfrac{-4}{x-2}$

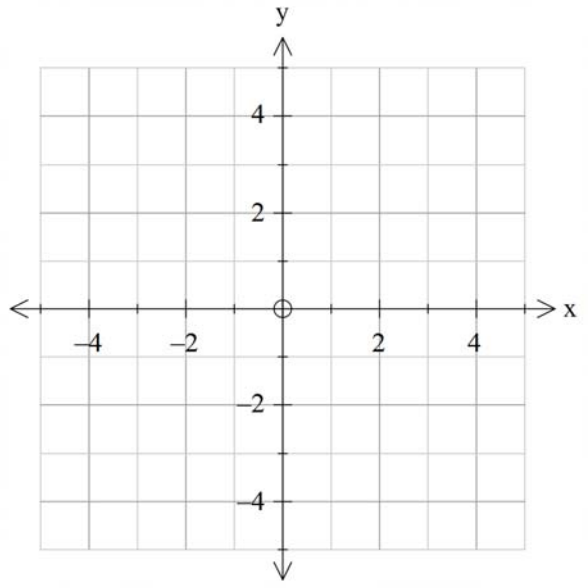
(a) State the equations of the vertical and horizontal asymptotes.

(2 marks)

(b) Find the y-intercept.

(1 mark)

(c) Draw the graph of this function on the axes provided showing clearly the information from parts (a) and (b).



(3 marks)

(d) (i) Discuss what happens to y as $x \to \infty$.

(1 mark)

(ii) Discuss what happens to y as $x \to -\infty$.

(1 mark)

QUESTION 4 **4 marks**

A life raft designed for 8 people has sufficient food and water to last 10 days.

(a) Is the relationship between the number of people and the days the food will last a direct or indirect relationship, giving a brief explanation for your answer.

(1 mark)

(b) If the number of people is represented by p and days the food will last by d, state the equation which relates p and d.

(1 mark)

The ship it was from sank so quickly that only 5 people made it to the raft.

(c) Using the equation written in (b) or otherwise, determine how many days the food will last these 5 people.

(2 marks)

QUESTION 5 **6 marks**

Consider the function $g(x) = 2 - \sqrt{x+1}$

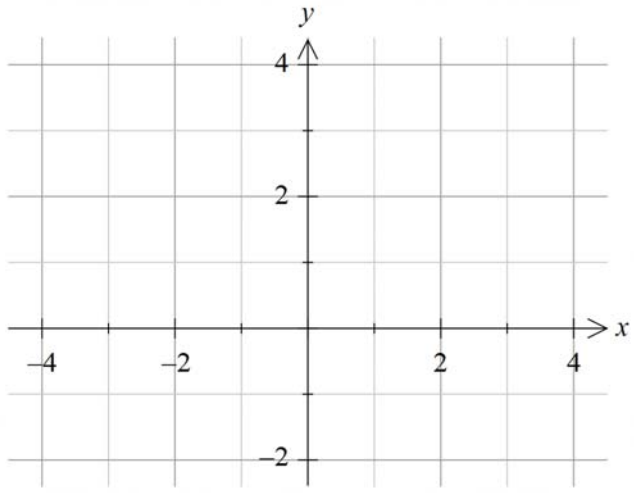
(a) State the Domain of $g(x) = 2 - \sqrt{x+1}$.

(1 mark)

(b) (i) Find the x and y intercept for $g(x) = 2 - \sqrt{x+1}$.

(2 marks)

(ii) On the set of axes below graph $g(x) = 2 - \sqrt{x+1}$ showing the information obtained in (i).



(2 marks)

(iii) Hence state the range of $g(x) = 2 - \sqrt{x+1}$.

(1 mark)

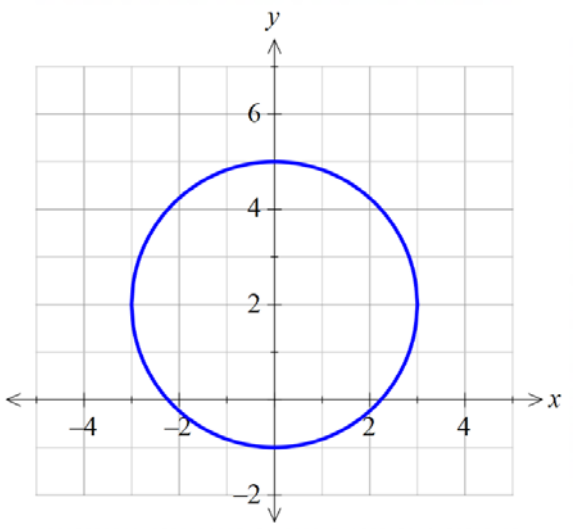
QUESTION 6 **7 marks**

For the following relations:

(i) State with reasons, whether or not it is a function;

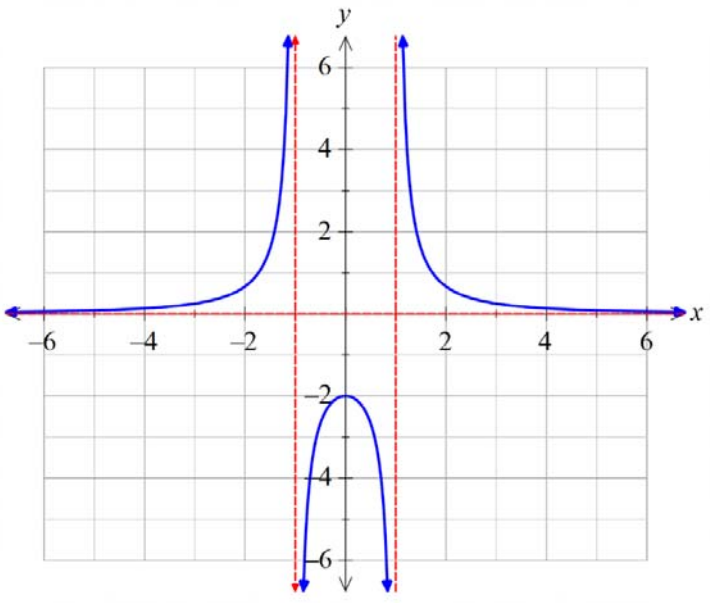
(ii) State the domain and range.

(a)



(3 marks)

(b)

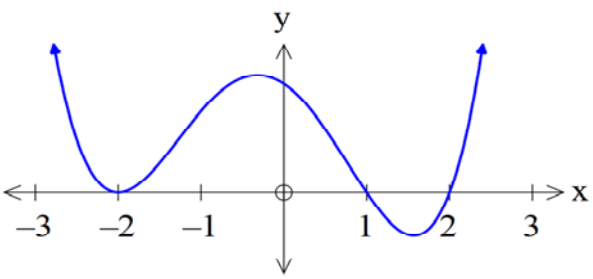


(4 marks)

QUESTION 7 **4 marks**

Construct sign diagrams for the following

(a)



(2 marks)

(b) $y = \dfrac{2x+1}{2-x}$

(2 marks)

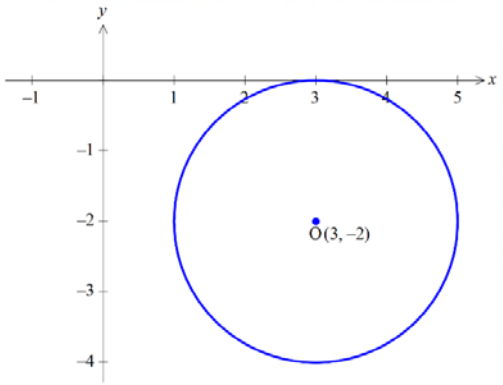
QUESTION 8 **4 marks**

Find the equation of the following circles in Centre-Radius form (i.e. $(x-h)^2 + (y-k)^2 = r^2$)

(a) Centre $(1, -3)$ radius = $2\sqrt{3}$ units

(2 marks)

(b)



(2 marks)

QUESTION 9 **7 marks**

(a) Express in expanded form (i.e $x^2 + y^2 + Ax + By + C = 0$):

$(x-3)^2 + (y-1)^2 = 81$

(2 marks)

(b) (i) Express in Centre-Radius form (i.e. $(x-h)^2 + (y-k)^2 = r^2$):

$x^2 + y^2 - 8x + 6y = 0$

(3 marks)

(ii) A Krispy Kreme shop has opened up recently and is located on a grid reference such that the shop is at the centre of the circle from (i) and delivery is within the radius found in (i). If Joseph lives at position $(1, -1)$, will he be able to get a Krispy Kreme delivered from this shop?

(2 marks)

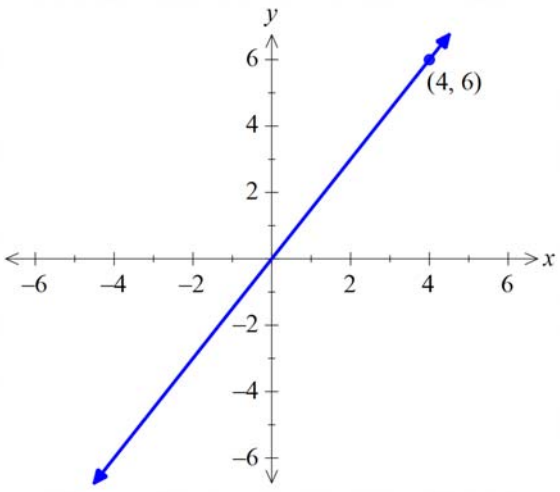
QUESTION 1 **4 marks**

(a) Consider the following graph.

(i) Explain why the variables are directly proportional.

Straight Line through the origin ✓ Straight Line through origin ✓

(ii) Determine the equation connecting the variables



$6 = 4 \times 1.5 \Rightarrow y = 1.5x$ ✓

(2 marks)

(b) Consider the following table.

(i) State whether the variables are directly or inversely proportional

Indirectly Proportional ✓

(ii) Determine the equation connecting the variables

a	-2	-1	1	2
y	3	6	-6	-3

$a \times y = -6$ ✓

(2 marks)

QUESTION 2 **8 marks**

If $f(x) = 5 - 2x$ and $g(x) = (x-1)^2$, find in simplest form

(a) $f(-1) = 5 - 2 \times (-1) = 7$ ✓

(1 mark)

(b) $g(f(3)) = g(5 - 2 \times 3) = g(-1)$ ✓ $= (-1-1)^2 = 4$ ✓

(2 marks)

(c) $f(3-x) = 5 - 2(3-x)$ ✓ $= -1 + 2x$ ✓

(2 marks)

(d) Solve for x : $f(f(x)) = 3$

$$f(f(x)) = 3$$
$$\therefore \quad f(5-2x) = 3$$
$$\therefore 5 - 2(5-2x) = 3 \quad ✓$$
$$\therefore \quad 5 - 10 + 4x = 3$$
$$\therefore \quad 4x - 5 = 3 \quad ✓$$
$$\therefore \quad 4x = 8$$
$$\therefore \quad x = 2 \quad ✓$$

(3 marks)

QUESTION 3 **8 marks**

Consider the function $y = \dfrac{-4}{x-2}$

(a) State the equations of the vertical and horizontal asymptotes.

Vertical Asymptote $x = 2$ ✓
Horizontal Asymptote $y = 0$ ✓

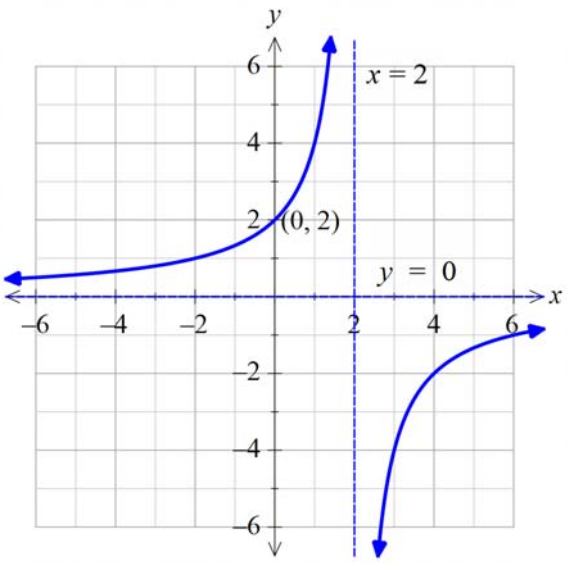
(2 marks)

(b) Find the y-intercept.

$x = 0 \Rightarrow y = \dfrac{-4}{-2} = 2$ ✓

(1 mark)

(c) Draw the graph of this function on the axes provided showing clearly the information from parts (a) and (b).



Asymptotes ✓
Y intercept ✓
Shape ✓

(3 marks)

(d) (i) Discuss what happens to y as $x \to \infty$.

As $x \to \infty$ $y \to 0^-$ ✓

(1 mark)

(ii) Discuss what happens to y as $x \to -\infty$.

As $x \to -\infty$ $y \to 0^+$ ✓

(1 mark)

QUESTION 4 **4 marks**

A life raft designed for 8 people has sufficient food and water to last 10 days.

(a) Is the relationship between the number of people and the days the food will last a direct or indirect relationship, giving a brief explanation for your answer.

Indirect relationship as less people means food will last longer ✓
So as people decreases days increases

(1 mark)

(b) If the number of people is represented by p and days the food will last by d, state the equation which relates p and d.

$pd = 8 \times 10 = 80$ ✓

(1 mark)

The ship it was from sank so quickly that only 5 people made it to the raft.

(c) Using the equation written in (b) or otherwise, determine how many days the food will last these 5 people.

$5d = 80$ ✓

$\therefore \quad d = 16$ ✓

(2 marks)

QUESTION 5 **6 marks**

Consider the function $g(x) = 2 - \sqrt{x+1}$

(a) State the Domain of $g(x) = 2 - \sqrt{x+1}$.

$x + 1 \geq 0 \Rightarrow x \geq -1$ ✓

(1 mark)

(b) (i) Find the x and y intercept for $g(x) = 2 - \sqrt{x+1}$.

x Intercept $y = 0 \Rightarrow 2 - \sqrt{x+1} = 0$

$\therefore \sqrt{x+1} = 2$

$\therefore \quad x + 1 = 4 \Rightarrow x = 3$ ✓ or use gcalc

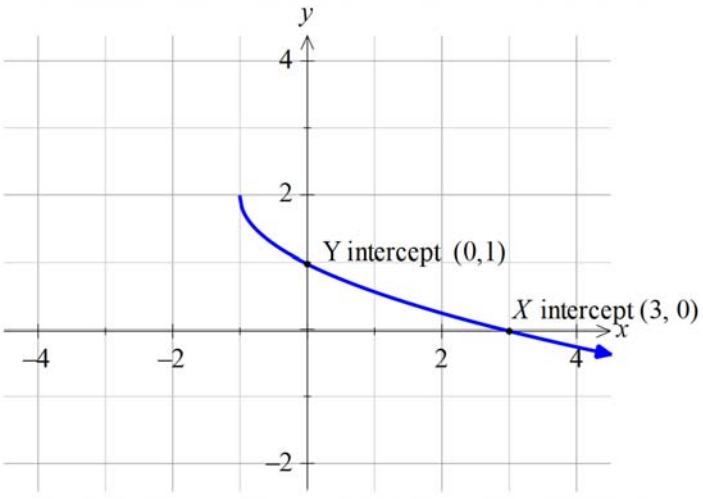
Y intercept $x = 0 \Rightarrow y = 2 - \sqrt{0+1} = 1$ ✓

(2 marks)

(ii) On the set of axes below graph $g(x) = 2 - \sqrt{x+1}$ showing the information obtained in (i).

X and Y intercepts correct ✓

General Shape ✓



(2 marks)

(iii) Hence state the range of $g(x) = 2 - \sqrt{x+1}$.

$y \leq 2$ ✓

(1 mark)

QUESTION 6 **7 marks**

For the following relations:

(i) State with reasons, whether or not it is a function;

(ii) State the domain and range.

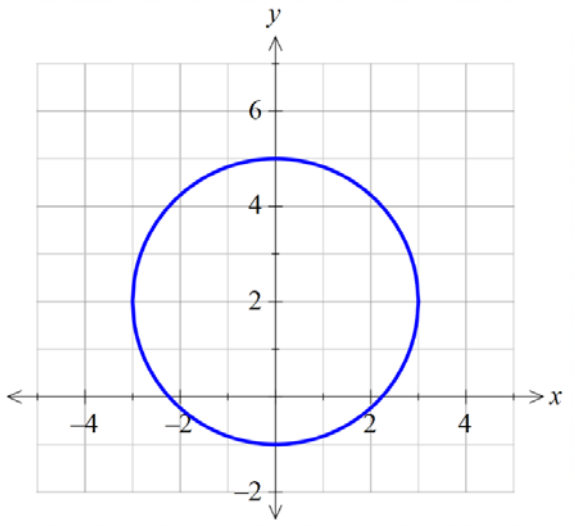
(a)

Doesn't pass vertical line test ✓

∴ Not function

Domain $-3 \leq x \leq 3$ ✓

Range $-1 \leq y \leq 5$ ✓



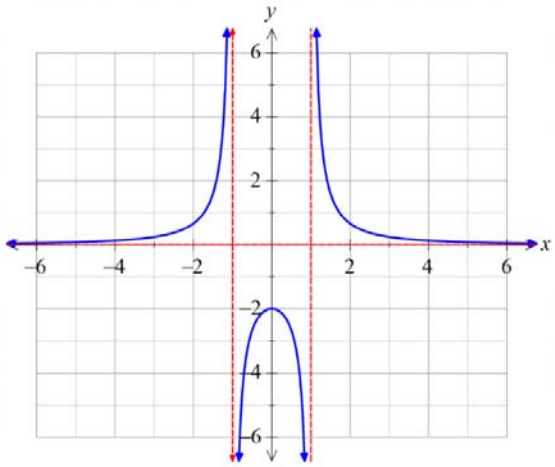
(3 marks)

(b)

Does pass vertical line test ⇒ Function ✓

Domain $x \neq -1,\ x \neq 1$ ✓

Range $y \leq -2$ ✓ or $y > 0$ ✓

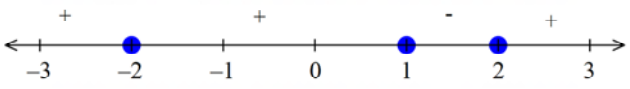


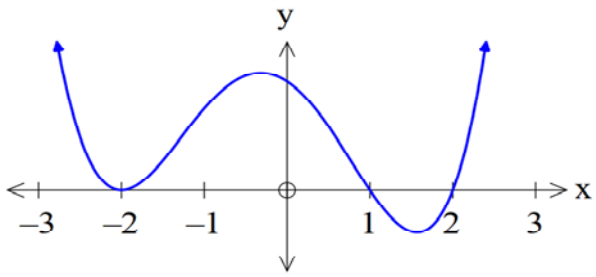
(4 marks)

QUESTION 7 **4 marks**

Construct sign diagrams for the following

(a)





Dots correct ✓

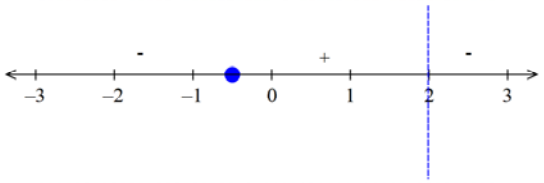
Signs correct ✓

(2 marks)

(b) $y = \dfrac{2x+1}{2-x}$

Dots and dotted line correct ✓

Signs correct ✓



(2 marks)

QUESTION 8 **4 marks**

Find the equation of the following circles in Centre-Radius form (i.e. $(x-h)^2 + (y-k)^2 = r^2$)

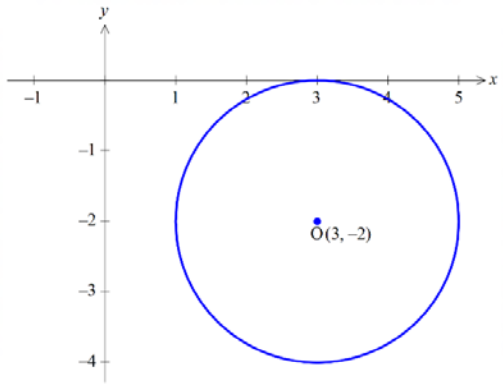
(a) Centre $(1, -3)$ radius = $2\sqrt{3}$ units

$(x-1)^2 + (y+3)^2$ ✓ $= (2\sqrt{3})^2 = 12$ ✓

(2 marks)

(b)

$(x-3)^2 + (y+2)^2$ ✓ $= 2^2 = 4$ ✓



(2 marks)

QUESTION 9 **7 marks**

(a) Express in expanded form (i.e $x^2 + y^2 + Ax + By + C = 0$):

$(x-3)^2 + (y-1)^2 = 81$

$x^2 - 6x + 9 + y^2 - 2y + 1 = 81$ ✓

$\therefore \quad x^2 + y^2 - 6x - 2y - 71 = 0$ ✓

(2 marks)

(b) (i) Express in Centre-Radius form (i.e. $(x-h)^2 + (y-k)^2 = r^2$):

$x^2 + y^2 - 8x + 6y = 0$

$(x^2 - 8x + 16) + (y^2 + 6y + 9) = 16 + 9$

$\therefore \quad (x-4)^2$ ✓ $+ (y+3)^2$ ✓ $= 25$ ✓

(3 marks)

(iii) A Krispy Kreme shop has opened up recently and is located on a grid reference such that the shop is at the centre of the circle from (i) and delivery is within the radius found in (i). If Joseph lives at position $(1, -1)$, will he be able to get a Krispy Kreme delivered from this shop?

Radius = 5

Distance from (4,-3) to (1,-1)

$= \sqrt{3^2 + 2^2} = \sqrt{13}$ ✓

Since $\sqrt{13} < 5 \Rightarrow$ will get delivery ✓

(2 marks)