

Essential Mathematics 2024

Question booklet

Topic 2: Measurement (Questions 1 to 3) 30 marks

Topic 4: Statistics (Questions 4 to 6) 30 marks

Topic 5: Investments and Ioans (Questions 7 to 9) 30 marks

- Answer all questions
- · Write your answers in this question booklet
- · You may write on pages 11, 17, and 23 if you need more space
- Allow approximately 40 minutes for each topic

Examination information

Materials

- Question booklet
- SACE registration number label

Instructions

- · Show appropriate working and steps of logic in this question booklet
- · Use black or blue pen
- · You may use a sharp dark pencil for diagrams and graphical representations
- · Approved calculators may be used complete the box below

Total time: 130 minutes Total marks: 90

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The SACE Board of South Australia acknowledges that this examination was created on Kaurna Land. We acknowledge First Nations Elders, parents, families, and communities as the first educators of their children, and we recognise and value the cultures and strengths that First Nations students bring to the classroom. We respect the unique connection and relationship that First Nations peoples have to Country, and their ever-enduring cultural heritage.

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| Attach your SACE registration number label here | Model | 40UZZ |
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Question 1 (11 marks)

A basketball stadium has two types of wall mounting brackets for basketball backboards, as shown in Image 1 and Image 2 below.

Both types of wall mounting brackets have a supporting wire that attaches the basketball backboards to the wall.



Image 2: Wall mounting bracket 2



Source: adapted from Grand Slam Sports Equipment, viewed 20 June 2024, www.grand-slam.com.au

Diagram 1 below shows measurements for wall mounting bracket 1. The supporting wire is CD.



Diagram 1

[This diagram is not drawn to scale.]

(a) (i) Show that the length of AB is 3 m.

(2 marks)

(ii) Calculate the length of the supporting wire CD.

(2 marks)

Question 1 continues on page 4.

Diagram 2 below shows measurements for wall mounting bracket 2.

The supporting wire is HJ.



Diagram 2

[This diagram is not drawn to scale.]

(b) (i) Show that angle HGF is 20.2° .

(3 marks)

(ii) Show that angle HGJ is 79.9° .



(1 mark)

(iii) Calculate the length of the supporting wire HJ.

(2 marks)

(c) The basketball stadium has a requirement that the supporting wire that attaches the basketball board to the wall should not be longer than 4 m in length.

Tick the appropriate box below to indicate whether the following statement is true or false.

The supporting wire for both types of wall mounting bracket meets the stadium requirement.

True

False

(1 mark)

Question 2 (7 marks)

Bunting can be made from individual flags held together with rope, as shown below.



Source: adapted from © _human | istockphoto.com

Diagram 3 below shows five identical flags on a $1\,\mathrm{m}$ length of rope.

There is a 1 cm gap between each flag and 8 cm at each end of the rope to tie the bunting on display.





[This diagram is not drawn to scale.]

The material for one flag consists of two shapes, a rectangle and a trapezium. Diagram 4 below shows the shapes and dimensions of one flag.



Diagram 4

[This diagram is not drawn to scale.]

(a) State whether or not it is suitable to have *six* identical flags on the 1 m length of rope. Justify your statement with a calculation.

(2 marks)

(b) Calculate the area of one flag.



(3 marks)

Identical flags, as shown in Diagram 4, are cut from a rectangular piece of material with measurements $75 \text{ cm} \times 20 \text{ cm}$ following the pattern shown in Diagram 5 below.



Diagram 5

(c) State whether one rectangular piece of material with measurements $75 \text{ cm} \times 20 \text{ cm}$ is suitable to make *five* identical flags, using the pattern shown in Diagram 5. Justify your answer with a calculation.

Question 3 (12 marks)

A child is using the toy digger shown below to remove sand from a sandbox.

This image has been removed for copyright reasons.



[These diagrams are not drawn to scale.]

The scoop of the digger is a prism, as shown in Diagram 6.

The uniform cross-section area of the scoop, as shown in Diagram 7, can be approximated as a semicircle with a diameter of $12\ {\rm cm}.$

(a) Calculate the approximate uniform cross-section area of the scoop.

(b) The uniform cross-section area of the scoop can be approximated using the measurements shown below in Diagram 8.



Diagram 8

[This diagram is not drawn to scale.]

(i) Using Simpson's rule, show that the approximate uniform cross-section area of the scoop is 42.2 cm^2 .

(3 marks)

(ii) Calculate the approximate volume of the scoop, using the area obtained in part (b)(i) and information in Diagram 6 on page 8.

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(1 mark)

(c) The child used the digger to remove the sand from the sandbox shown below.



(i) Determine the maximum number of *full scoops* of sand that can be removed using the scoop on the digger.

(3 marks)

(ii) State one assumption you made in your answer to part (c)(i).

(1 mark)

(d) The sand in this sandbox has a density of 1.6 grams per cubic centimetre (g/cm³). Determine the approximate mass (in kilograms) of the sand in the sandbox.

You may write on this page if you need more space to finish your answers to any questions in Topic 2. Make sure to label each answer carefully (e.g. 3(b)(i) continued).

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Question 4 (12 marks)

A learner driver in South Australia must complete at least 75 hours of supervised driving before obtaining their provisional licence, also known as 'P plates'.

The back-to-back stem-and-leaf diagram below shows the number of supervised driving sessions taken to complete the minimum 75 hours by 15 learner drivers in metropolitan Adelaide and by 15 learner drivers in Port Augusta.

| Numb | ber of su me | upervise tropolite | ed drivii an Adela | ng sess aide | ions – | Stem | Numb | per of si | upervise Port A | ed drivir ugusta | ng sessi | ions — |
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Key: 9 | 1 = 91

(a) (i) Complete Table 1 below.

Table 1

| Statistical measure | Supervised driving sessions – metropolitan Adelaide | Supervised driving sessions – Port Augusta |
|---------------------|--|---|
| mean | 110 | |
| standard deviation | 14.2 | 20.5 |
| minimum | | 72 |
| lower quartile | 98 | |
| median | 111 | |
| upper quartile | 122 | |
| maximum | | 149 |

(3 marks)

(ii) It has been stated that learner drivers in metropolitan Adelaide typically do more supervised driving sessions to complete the minimum 75 hours than learner drivers in regional cities such as Port Augusta.

Using an appropriate statistical measure from Table 1, explain whether or not this statement is supported.

(b) (i) Compare the variability of the number of supervised driving sessions taken to complete minimum 75 hours in these two locations. Use an appropriate statistical measure from Table 1 on page 12 to justify your comparison.

(2 marks)

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(c) (i) *Complete and label* the box-and-whisker diagrams below for the metropolitan Adelaide and Port Augusta data.

(2 marks)



Supervised driving sessions

- (ii) Using the box-and-whisker diagrams above, complete the following sentences:
 - (1) _____% of learner drivers in metropolitan Adelaide had between 98 and 122 supervised driving sessions.
 - (2) All of the learner drivers in metropolitan Adelaide required more supervised driving sessions than ______% of learner drivers in Port Augusta.

Question 5 (12 marks)

A study was conducted to find the most cost-effective sources of protein that can be included in a healthy diet. Table 2 below shows the protein content (%) and cost (\$ per kilogram) of 10 different foods.

| Food type | Protein content (%) | Cost (\$ per kilogram) |
|-----------|---------------------|------------------------|
| Peanuts | 24 | 7 |
| Beef | 26 | 16 |
| Lobster | 20 | 75 |
| Yoghurt | 10 | 6 |
| Tuna | 26 | 12 |
| Lentils | 9 | 5 |
| Turkey | 29 | 18 |
| Salmon | 23 | 40 |
| Milk | 3 | 3 |
| Chicken | 31 | 14 |

Table 2



Protein content and cost of different foods



(1 mark)

(b) Calculate the coefficient of determination (r^2) and state the strength of the relationship between protein content and the cost per kilogram.

(c) (i) Identify two food types that appear to be outliers in the data.

(2 marks)

(ii) Assume it is appropriate to remove these *two* outliers identified from the data.

Calculate the equation of the least squares regression line (line of best fit) after removing the *two* outliers from the data.

Record the equation in the box below.



(2 marks)

(iii) Using the equation calculated in (c)(ii), predict:

(1) the cost (\$ per kilogram) of protein powder that has a protein content of 78%.

(1 mark)

(2) the protein content (%) of tofu that has a cost of \$7 per kilogram.

(2 marks)

(d) Explain *one* factor that may affect the reliability of the predictions made in part (c)(iii).

Question 6 (6 marks)

Some federal politicians have called for a popular social media platform to be reviewed due to concerns about data privacy. Most users of this platform are in the age group of 15 to 34 years.

A research company decides to survey 6000 people from three Australian states about this concern. The states selected for the survey are Victoria, South Australia, and Western Australia.

The table below shows the approximate population of these states.

| Table 3: State | e population | (million | people) |
|----------------|--------------|----------|---------|
|----------------|--------------|----------|---------|

| Victoria | 6.7 |
|-------------------|-----|
| South Australia | 1.9 |
| Western Australia | 2.9 |

(a) Calculate how many participants should be surveyed from South Australia.

⁽² marks)

- (b) The research company is considering a self-selected survey method that is available on the popular social media platform.
 - (i) Explain *one advantage* of using this survey method to collect data about this concern.

(2 marks)

(ii) Explain *one disadvantage* of using this survey method to collect data about this concern.

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You may write on this page if you need more space to finish your answers to questions on Topic 4. Make sure to label each answer carefully (e.g.4(b)(i) continued).

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Question 7 (12 marks)

Michael plans to save for 5 years to purchase a caravan.

He is considering two investment accounts.

- **Bank A account:** offers an interest rate of 5.20% per annum, compounded quarterly.
- **Bank B account:** offers an interest rate of 5.05% per annum, compounded monthly.

The caravan Michael plans to purchase costs \$94265.

(a) Show that the quarterly payments required into the Bank A account to save \$94265 in 5 years is approximately \$4160.



(2 marks)

(b) (i) Michael can only afford to save \$4160 each quarter. Show that the equivalent monthly payment would be approximately \$1390 per month.

(1 mark)

(ii) Calculate how long (in months) it will take for Michael to reach his goal of \$94265 if he deposits the equivalent monthly payment into the Bank B account.

(c) State why the time to save the \$94265 in the two accounts is the same even though their interest rates are different.

(1 mark)

Bank C offers an interest rate of 5.25% per annum, compounded monthly. This account requires an opening balance of \$2000.

(d) (i) Calculate the monthly payment required into the Bank C account for Michael to save \$94265 to purchase the caravan in 5 years' time.

(2 marks)

(ii) Calculate the total interest earned from Bank C over the 5 years.

(2 marks)

Michael realised that the cost of caravans will increase over time. Therefore, he will need to save more money to purchase a similar caravan in 5 years' time.

(e) The cost of caravans is expected to increase with inflation at 3.95% per annum. Calculate the predicted cost of a similar caravan in 5 years' time.

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Question 8 (12 marks)

Derek is a first home buyer who needs to borrow \$425000 to purchase his new house.

(a) Show that Derek's loan interest rate is 4.94% per annum if his minimum monthly repayments are \$2265 and the term of the loan is 30 years.

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(2 marks)

(b) (i) Calculate the time (in months) to repay the loan if Derek makes increased repayments of \$2800 from the beginning of the loan. Assume the interest rate remains at 4.94% per annum, compounded monthly.

(2 marks)

(ii) Calculate the interest saved by making the increased repayments.

Derek is only able to make the increased repayments of \$2800 for the first 5 years. Assume the interest rate remains at 4.94% per annum, compounded monthly, and the amount borrowed remains the same.

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- (c) (i) Calculate the outstanding debt after 5 years if he pays the increased repayments. Assume the loan rate and amount borrowed remain the same.

After 5 years, Derek can only afford the minimum monthly repayments of \$2265.

(ii) Calculate the remaining time (in years) to repay the loan if Derek makes the minimum monthly repayments after the 5 years.

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⁽² marks)

- (d) Consider the following:
 - **Scenario 1:** Derek makes the increased monthly repayments in the first 5 years of the loan, as calculated in part (c).
 - **Scenario 2:** Derek makes the increased monthly repayments for 5 years after 10 years of the loan term had passed.

Explain whether Derek would take more or less time to repay the loan if he is making increased payments under Scenario 2 compared to making increased payments under Scenario 1.

Question 9 (6 marks)

Sandra earns \$63 600 per year working as a pharmacist.

Her superannuation fund has an average return rate of 7.3% per annum, compounded quarterly, and her employer contributes quarterly payments of 11% of her salary into her superannuation fund.

(a) Show that the employer quarterly contribution is \$1749.

(1 mark)

(b) Calculate Sandra's superannuation balance at age 65 if she began working at age 25. Assume the employer quarterly contributions and the fund return rate remain constant over her working life.

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(2 marks)

Sandra had a goal of retiring with 2000000 and realises that she will need to make voluntary contributions in addition to the employer contributions.

(c) Calculate the quarterly voluntary contribution that Sandra would need to make to achieve her retirement goal. Assume all other superannuation conditions remain the same.

(2 marks)

(d) Other than making voluntary contributions, state *one* way Sandra could reach her goal of retiring with \$2 000 000.

You may write on this page if you need more space to finish your answers to questions on Topic 5. Make sure to label each answer carefully (e.g. Question 8(b)(ii) continued).