

General Mathematics 2024

Question booklet

- Questions 1 to 8 (90 marks)
- Answer **all** questions
- · Write your answers in this question booklet
- · You may write on pages 13 and 24 if you need more space

Examination information

Materials

- Question booklet
- SACE registration number label

Instructions

- · Show appropriate working and steps of logic in this question booklet
- Use black or blue pen
- · You may use a sharp dark pencil for diagrams and graphical representations
- Approved calculators may be used complete the box below

Total time: 130 minutes Total marks: 90

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The SACE Board of South Australia acknowledges that this examination was created on Kaurna Land. We acknowledge First Nations Elders, parents, families, and communities as the first educators of their children, and we recognise and value the cultures and strengths that First Nations students bring to the classroom. We respect the unique connection and relationship that First Nations peoples have to Country, and their ever-enduring cultural heritage.

	Graphics calculator
	1. Brand
Attach your SACE registration number label here	Model
	2. Brand
	Model Government of South Austral



Question 1 (10 marks)

A dog grooming salon has four staff members. To groom a dog, the staff wash, trim, and dry and brush each dog's hair. The staff also clip each dog's nails and clean their teeth. The manager wants to determine which task each staff member should undertake daily in order to minimise the total completion time to groom a dog.

Table 1 below shows the average time spent (in minutes) by each staff member to complete each task.

	Wash	Trim	Dry and Brush	Nails and Teeth
Delilah	33	40	36	18
Owen	33	50	25	27
Gracie	32	53	40	22
Zephyr	35	50	26	30

Table 1

(a) Interpret the meaning of the number '25', shaded in Table 1 above.

(1 mark)

The Hungarian algorithm has been applied to the information from Table 1 shown above, resulting in the following reduced array:

7	0	18	0
0	3	0	2
2	9	18	0
1	2	0	4

(b) Using the reduced array, *complete* Table 2 below, showing which task should be allocated to each employee to minimise the total completion time to groom a dog. Assume that each staff member will be allocated one task.

	Task	Time taken
Delilah		
Owen		
Gracie		
Zephyr		
Тс	tal time	

Table 2

The manager would like to launch a new advertising campaign that guarantees 'to groom a dog in under two hours, or the next groom is free'.

The manager wants to investigate if allocating more than one job to Gracie would allow them to use this campaign.

Table 3 below shows the average time spent (in minutes) by each staff member to complete each task, with an additional row when Gracie is allocated one more job.

	Wash	Trim	Dry and Brush	Nails and Teeth
Delilah	33	40	36	18
Owen	33	50	25	27
Gracie	32	53	40	22
Gracie	32	53	40	22
Zephyr	35	50	26	30

Table 3

A new array to represent this new information from Table 3 is shown below.

33	40	36	18	0
33	50	25	27	0
32	53	40	22	0
32	53	40	22	0
35	50	26	30	0

(c) State why the new array must have an additional column of zeros.

(1 mark)

After completion of the first two steps of the Hungarian algorithm, the following array is achieved:

2	0	11	0	1
2	10	0	9	1
0	12	14	3	0
0	12	14	3	0
3	9	0	11	0

(d) Draw lines on the array above to show that the optimal solution has not yet been achieved.

2		0	11	0	1												
2	-	10	0		1	_				 	 					 	
	-			-	_	_			 	 	 	 	 		 		
0	-	12	14		0	_											
0	1	12	14	3	0												
3		9	0	11	0												
				_					 	 							

(e) (i) Complete the remaining steps of the Hungarian algorithm, **and** determine the updated minimum completion time to groom a dog.

(3 marks)

(ii) Which staff member is *no* longer required to complete a task?

(1 mark)

(f) State why the dog grooming salon guaranteeing customers 'to groom a dog in under two hours, or the next groom is free' should be considered unreasonable.

Question 2 (10 marks)

The age at which Australian children start walking is used as a marker for childhood development. The data collected is normally distributed with a mean of 14 months and a standard deviation of 1.75 months.

(a) Calculate the proportion of children that starts walking between the ages of 10.5 months and 14 months.

(1 mark)

Federal government health guidelines recommend that state governments offer funding assistance if a child is not walking by the age of 17 months.

(b) (i) Calculate the percentage of children that may require funding assistance with walking.

(1 mark)

(ii) In South Australia, 19 502 births were registered in 2022. How many of these children are expected to require funding assistance with walking?

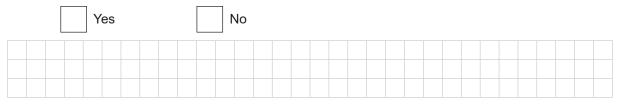
(1 mark)

The state government health authority provides funding assistance to the oldest 10% of children not yet walking.

(c) Calculate the minimum age that a child would need to be in order to get this state government funding assistance.

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(d) Is the state government providing enough funding assistance according to the federal government recommendations? **Tick the appropriate box** to indicate your answer, *and* **justify your answer** by making reference to your solution(s) from parts (b) and/or (c).

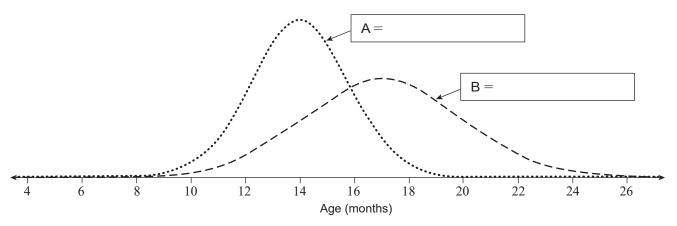


(1 mark)

Two other markers for childhood development are the age at which children start to speak and when their first molar tooth eruption occurs.

- Speaking age is normally distributed with a mean of 17 months and standard deviation of 3 months
- First molar tooth eruption is normally distributed with a mean age of 14 months and a standard deviation of 3 months.

The diagram below shows the probability distributions for *two of the three* markers for childhood development.



- (e) (i) Fill in boxes A and B to identify the markers for childhood development shown on the diagram above. (1 mark)
 - (ii) Using the information provided throughout this question, *draw* the missing probability distribution on the diagram above. (1 mark)

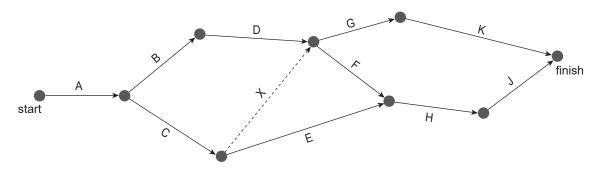
Federal government health guidelines recommend that children who are not speaking by the age of 24 months should start receiving funding assistance with speaking.

(f) Is a child more likely to require funding assistance with walking or speaking? Support your answer using appropriate calculations and reasoning.

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Question 3 (8 marks)

Peta is considering selling her house and moving to a larger home to accommodate her growing family. The network diagram below shows a series of tasks that need to be completed for this to happen.



(a) Explain why the dummy link marked X is required in the network diagram.

(2 marks)

The following precedence table has been completed for the above network diagram:

Task	Prerequisite task/s	Time to complete (days)	EST (days)	LST (days)
А	-	5	0	0
В	А	1	5	7
С	А	4	5	5
D	В	2	6	8
E	С	2	9	9
F	C, D	1	9	10
G	C, D	2	9	12
Н	E, F	3	11	11
J	G, H	3	14	14
К	G	2	11	15

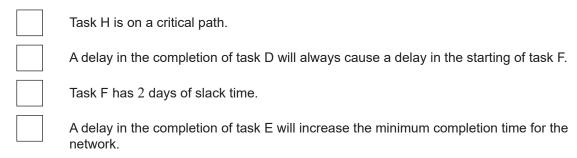
Table 4

(b) The network diagram shown above should contain two dummy links.

Using the information from the precedence table, *draw* the second dummy link on the network diagram above.

(c) (i) Using the information from the precedence table and the network diagram, identify the statements that are correct.

Place a tick in the boxes next to all correct statements below.
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(2 marks)

(ii) State the critical path **and** the minimum completion time for this network.

Question 4 (17 marks)

Nia has started her first job, earning \$67500 per annum. Nia's employer will contribute 11.5% of her salary into her superannuation fund, and Nia plans to contribute an extra 1% of her salary into the same account.

Nia's superannuation account earns interest at 7.2% per annum, compounded fortnightly.

(a) (i) Show that approximately \$325 per fortnight is deposited into Nia's superannuation account.

(1 mark)

(ii) Calculate the balance in Nia's superannuation account after 40 years of Nia working in this job.

(2 marks)

(iii) Calculate the amount of interest that Nia's superannuation account will earn over the 40 years.

(1 mark)

(b) (i) State *one* reason why the balance in Nia's superannuation account could be lower than the balance calculated in part (a)(ii).

(1 mark)

(ii) Hence, describe a strategy that Nia could implement to achieve the balance calculated in part (a)(ii).

At retirement, Nia deposits 2000000 into an annuity account that has an interest rate of 4.6% per annum, compounded weekly. She wishes to withdraw a weekly income from this account for 30 years.

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(c) (i) Show that Nia can withdraw a weekly income of approximately \$2365.

(2 marks)

(ii) Calculate the time taken in years for the balance of the annuity account to reach \$1000000.

(2 marks)

(iii) Using mathematical reasoning, explain why your answer to part (c)(ii) is not halfway through the expected 30-year term of the annuity account.

(2 marks)

Question 4 continues on page 12.

When Nia retires, her initial cost of living is 1000 per week. She would like to maintain her standard of living throughout her retirement.

At any time throughout her retirement, Nia's weekly cost of living (C) can be predicted using the following equation, where t is the time in years:

$$C = 1000 \times (1.031)^t$$

(d) (i) Complete the following statement to best explain what the value 1.031 represents in the context of this scenario.

The value 1.031 indicates that the cost of living is predicted to _____

by ______% per annum. The term used to describe this measure is ____

(2 marks)

(ii) Using the equation from part (d)(i), predict Nia's weekly cost of living exactly 5 years into her retirement.

(1 mark)

(iii) Predict the time in years it would take for Nia's cost of living to reach \$2365 per week.

(1 mark)

(e) Using evidence from parts (c) and (d), provide *one* reason why it could be considered reasonable for Nia to maintain her standard of living throughout her 30 years of retirement.



You may write on this page if you need more space to finish your answers. Make sure to label each answer carefully (e.g. 4(b)(ii) continued).

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Question 5 (9 marks)

The Australian Veterinarian Association recommended that owners take their cat to visit a vet annually. Owners following this advice can expect the total cumulative costs for their cat's annual visit to a vet over its lifetime to be as per Table 5 below.

Table 5

Cat age in years (t)	0	2	4	6	8	10	12
Cumulative vet costs in (C)	1200	1330	1680	1775	2150	2300	2500

Table 6

Regression equation	Coefficient of determination (r^2)
C = 113t + 1172	0.986
$C = 1224 \times (1.065)^t$	0.975

Based on the information above, the linear model is the best model for predicting cumulative vet costs when the owners follow the recommended annual visit to a vet.

(a) Using the linear model stated in Table 6, predict the cumulative vet costs of a 9-year-old cat.

(1 mark)

If a cat does not visit a vet annually as recommended, the total cumulative vet costs for a cat over its lifetime are shown below in Table 7.

Table 7

Cat age in years (t)	0	2	4	6	8	10	12
Cumulative vet costs in \$ (C)	400	850	1700	3650	7200	11 200	14 500

Table 8

Regression equation	Coefficient of determination (r^2)
	0.922
$C = 475 \times (1.36)^t$	0.980

(b) (i) Calculate the linear regression equation for a cat that does not visit a vet annually, and **write the equation in Table 8 above.** (1 mark)

(ii) State why the exponential model $C = 475 \times (1.36)^t$ would better model the data from Table 7.

(iii) Interpret the meaning of the a value from the exponential model shown in Table 8 in the context of the question.

(1 mark)

(iv) Using the equation for the exponential model from Table 8, predict the total cumulative vet costs of a 15-year-old cat that does not visit the vet annually.

(1 mark)

(v) Comment on the reasonableness of your solution to part (b)(iv).

(1 mark)

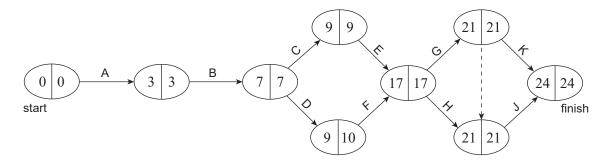
(c) (i) Using technology or otherwise, find the coordinate of the point of intersection for the linear model C = 113t + 1172 and the exponential model $C = 475 \times (1.36)^t$.

(1 mark)

(ii) Interpret your result from part (c)(i) in the context of the question.

Question 6 (11 marks)

For the upcoming Tay La concert, ticket agency A proposed the ticketing process represented by the network diagram below. The times for each task are given in weeks.



(a) (i) State the time taken to complete task B.

(1 mark)

(ii) State the time taken to complete task F.

(1 mark)

Task H has 1 week of slack time.

(b) (i) State how long it takes to complete task H.

(1 mark)

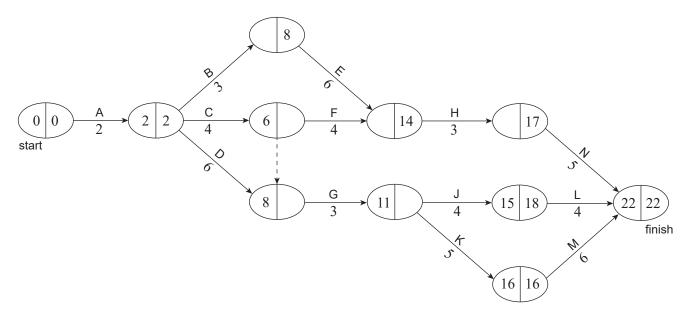
(ii) State the critical path(s) for this network.

(2 marks)

(c) State why task A and task B must always lie on **all** critical paths.

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A rival ticketing agency, ticket agency B, has submitted an alternative plan for Tay La to consider. The forward and backward scans have been partially completed in the network diagram below.



(d) (i) On the network diagram above, complete the **forward scan** by filling in the *three* missing values.

(1 mark)

(ii) On the network diagram above, complete the **backward scan** by filling in the *three* missing values.

(1 mark)

(1 mark)

Ticket agency B has underestimated the time taken to complete task J by 4 weeks.

(e) State the latest starting time for task B.

(f) Given this new information, provide mathematical evidence to show which ticket agency Tay La should select to provide the fastest ticketing process.

Question 7 (10 marks)

A study was conducted to investigate whether there is a relationship between public health expenditure of a country and the happiness of its citizens.

The data was measured as:

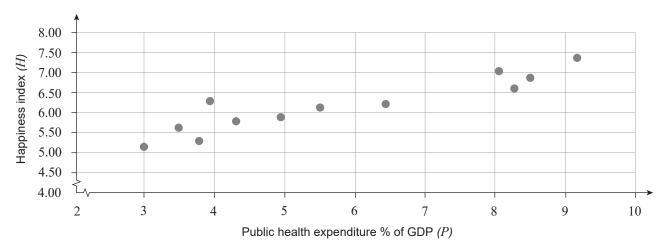
- Public Health Expenditure (P): the amount of money the country spent as a percentage of their gross domestic product (GDP).
- Happiness Index (*H*): a score out of 10 measuring general life satisfaction and happiness of the country's citizens.

The data for a group of randomly selected countries is shown below in Table 9.

This table has been removed for copyright reasons.

Source: adapted from 'World happiness report 2024', University of Oxford, Wellbeing Research Centre, viewed 25 July 2024, worldhappiness.report/data/; 'Healthcare spending', viewed 25 July 2024, ourworldindata.org

A scatterplot displaying the data is shown below.

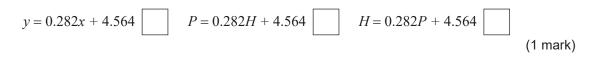


Happiness index v. Public health expenditure % of GDP

page **18** of **24**

(a) State the value of Pearson's Correlation coefficient (r) for a linear model.

(b) (i) Using correct variables, *tick* the model which best represents the equation of the least squares regression line for the data from Table 9.

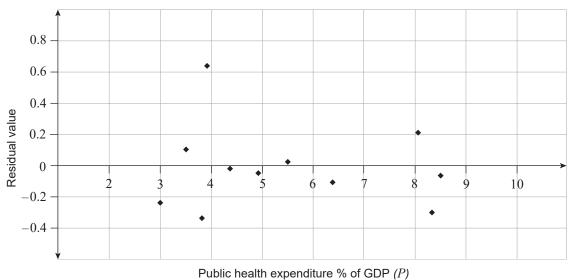


(ii) In the context of the question, interpret the meaning of 4.564 from the equation you selected in part (b)(i).

(1 mark)

(1 mark)

The residual plot for the linear model is shown below, with *one* missing residual value.



Residual plot for linear model

(c) On the graph above, add the missing residual value.

(1 mark)

(d) Using your answer to part (a) and the residual plot above, give *two* reasons why a linear model would be considered appropriate for this data.

India recorded a Happiness Index of 4.02.

(e) (i)	Using the linear regression equation you selected in part (b)(i), predict the percentage of GDP
	that India spent on public healthcare.

(1 mark)

(ii) In the context of the question, comment on the reasonableness of your solution to part (e)(i).

(1 mark)

Finland spent 7.3% of its GDP on public healthcare with a Happiness Index of 7.8.

(f) Using the equation you selected in part (b)(i), calculate the residual value for Finland.

Question 8 (15 marks)

Janet and Bill are twins who have just sold their food truck business. After all expenses have been repaid, they have each received a payout of \$12000.

Janet has decided to invest her \$12000 payout with her bank.

- (a) Janet's bank advertised an interest rate of 4% per annum; however, some information is missing from the billboard that she has seen on the bus stop.
 - (i) Complete Table 10 by calculating the missing effective rate.

Compounding period	Effective interest rate (%) per annum
Annually	4.000
Half-yearly	
Quarterly	4.060
Monthly	4.074
Weekly	4.079

Table 10

(1 mark)

(ii) Hence, state which compounding period option would be most financially beneficial for Janet.

(1 mark)

Janet has also seen a 4.5% flat rate investment option advertised on a billboard.

Table 11 shows the effective rate for this investment over several different lengths of time.

Table 11

Investment time length (years)	Effective interest rate (%) per annum
3	4.311
5	4.142
7	3.989
10	

(b) Complete Table 11 by calculating the missing effective rate.

Janet will invest the $$12\,000$ for 5 years. She has been given two options.

- Option A Interest compounding monthly at 4% per annum.
- Option B Flat rate of 4.5% per annum.

(c) Show that she will earn approximately \$50 more interest using Option B.

(3 marks)

Bill took out a home loan 6 years ago to purchase his first property. He borrowed $$510\,000$ from a bank offering an interest rate of 6.85% per annum, compounded monthly for 30 years. Bill makes the minimum monthly repayment of \$3341.82 for the duration of the loan.

(d) (i) Show that the outstanding balance of Bill's home loan after 6 years is approximately \$472000.

(2 marks)

(ii) Calculate the interest paid in the first 6 years of the loan.

(1 mark)

At this time, exactly 6 years into the loan, Bill decides to deposit his $12\,000$ payout into an offset account linked to his loan.

(e) (i) State the new balance on which interest will be charged.

(ii) Calculate the outstanding loan balance if Bill leaves his $12\,000$ in the offset account for 10 years. Assume that all other account conditions remain unchanged.

(1 mark)

After exactly 10 years of the $$12\,000$ being in the offset account, Bill withdraws $$7\,000$ from the offset account to pay for some home renovations.

(f) Calculate the total interest that Bill will save over the term of his loan by utilising the offset account.



(3 marks)

Janet and Bill have used their payout of \$12000 differently.

(g) State why Janet would need to consider tax implications, whereas Bill would not.

You may write on this page if you need more space to finish your answers. Make sure to label each answer carefully (e.g. 6(b)(ii) continued).

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