# Government of South Australia LogoSACE Board Logo2023 Specialist Mathematics Subject Assessment Advice

Overview

Subject assessment advice, based on the 2023 assessment cycle, gives an overview of how students performed in their school and external assessments in relation to the learning requirements, assessment design criteria, and performance standards set out in the relevant subject outline. They provide information and advice regarding the assessment types, the application of the performance standards in school and external assessments, and the quality of student performance.

Teachers should refer to the subject outline for specifications on content and learning requirements, and to the subject operational information for operational matters and key dates.

School Assessment

Overall Comments

Teachers can improve the moderation process and the online process by:

* thoroughly checking that all grades entered in Schools Online are correct. Errors in entered grades cannot usually be fixed through the moderation process, particularly if the error means a change in the rank order of results
* ensuring the uploaded tasks are legible, all facing up (and the same way), and have blank pages, student notes, and formula pages removed
* ensuring the uploaded tasks have pages the same size and in colour so teacher marking and comments are clearly distinguishable from student work
* using the same tasks; where possible, when combining with another school or schools to ensure standards are equitable. When combining classes across schools, teachers should be involved in moderation activities prior to uploading materials for the actual moderation to ensure that the rank order of the students within the combined assessment group is appropriate.

SAT Comments

Teachers can improve the moderation process and the online process by:

* ensuring the uploaded student SATs have been clearly marked. Showing which mathematical calculations are fully or partially correct and which are incorrect is a requirement of moderation. Showing marks and totals is also helpful
* preferably providing a summary of student results in each of the SATs at the start of the uploaded SATs file
* uploading the SATs as a single scanned file rather than six separate files to improve efficiency of the moderation process.

Investigation Comments

Teachers can improve the moderation process and the online process by:

* for investigations, comments and clearly marked mathematical calculations are a requirement of moderation
* ensuring uploaded investigations are the final work and not the draft. However, a draft can be assessed and uploaded if a student does not submit a final response

Assessment Type 1: Skills and Applications Tasks (50%)

Students complete six skills and applications tasks. Skills and applications tasks are completed under the direct supervision of the teacher. The equivalent of one skills and applications task must be undertaken without the use of either a calculator or notes.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:

* concepts and techniques
* reasoning and communication.

Teachers can elicit more successful responses by:

* ensuring the SAT is at a good level that allows for both routine questions and questions that include enough complexity to allow achievement at the higher-grade bands
* ensuring students have sufficient time to consider their responses for more complex questions as well as routine ones
* designing some complex questions to allow students to progress one step at a time through a process, using the ‘Show that …’ style of question
* structuring questions with multiple parts that begin with ‘access’ points to elicit C grade evidence and subsequently increase in complexity, with the potential to elicit A grade evidence
* providing a marks scheme and working space reflective of the cognitive demand of the question and showing the marks attained rather than a system of ticks and crosses which is often not clear to moderators
* providing students with appropriate feedback, including marks, to help them improve their work
* assessing conjecture and proof through this assessment type as they can be difficult to assess within a mathematical investigation
* ensure that your LAP accurately indicates where RC5 is being assessed
* keeping in mind that induction, per se, is not RC5, and completing proofs using mathematical induction does not achieve RC5 on its own. Students must be given the opportunity to form their OWN conjecture and then prove it. Teachers may choose to assess this in the Induction SAT, for example, by including a question of the form:

(a) Given the matrix , find (i)  (ii)  (iii) 

(b) On the basis of your answers to (a), make a conjecture about the matrix 

(c) Using the principle of mathematical induction, prove your conjecture for all positive integers 

* referring closely to the key questions and key concepts in the Subject Outline when designing assessment tasks
* while including material that is outside the subject outline can be considered an extension (e.g. Inequalities in Induction, Summation and Product notation in Induction, Euler’s form, or exponential form of a complex number and more complex integration by substitution), it should not be included to the detriment of including content required to be known, such as polar form in the summative SAT for example
* setting a variety of SATs that include limited questions drawn directly from past examinations. Schools can use a mid-year examination to provide students with a formative experience of examination style assessment; however, it is recommended that examinations are not included among the Assessment Type 1 SATs
* not marking crossed out work, as work the student has crossed out will not be marked in the final examination
* preferably not awarding half-marks as these are not awarded in the final examination and can inflate results and student expectations
* making students fully aware of the capabilities of their grahics calculator so they can make informed choices as to when and where to use it in completing SATs, particularly in graph work
* providing clear and accurate feedback on the appropriate use of mathematical notation, with particular attention needed for questions using vectors and integration which were identified as more common problem areas during moderation and examination marking.

The more successful responses commonly:

* provided clear and logical reasoning with correct mathematical notation
* displayed evidence that aligned with the question requirements (e.g. hence, exact)
* provided solutions that were efficient and demonstrated clear, logical and comprehensive understanding and interpretation of the question/problem
* used both algebraic and geometric approaches to solve problems in the complex numbers and 3D vectors topics
* showed all algebraic working by providing all relevant steps, particularly for the ‘Show that …’ style of question
* stated any theorems and/or properties that were being applied to support answers
* used mathematically-correct notation, particularly in questions using vectors and integration
* used clear labelling when multiple graphs on same set of axes was required
* labelled axes and scales of graphs correctly and indicated in Argand diagrams when vectors drawn were equal in length or perpendicular
* used the graphics calculator efficiently to draw both cartesian and parametric functions by plotting sufficient points, paying attention to correctly labelling and representing asymptotes, and correctly showing shape and behaviour of curves near asymptotes
* paid close attention to all details given in questions and the detail required in answering by showing conceptual thinking in their responses, no matter how simple
* included appropriate steps in applying algorithms and did not miss vital steps, especially in ‘Show that…’ questions where the answer is given in the question.

The less successful responses commonly:

* often did not attempt to answer questions, particularly more complex-style questions
* displayed incorrect or inconsistent mathematical notation and/or limited communication of reasoning i.e. the solution did not successfully ‘flow’ to a logical end
* provided unclearly-labelled graphs and incorrect scales
* included many arithmetic and algebraic mistakes that complicated the nature of the solutions (e.g. an error causing the student to have polynomials that did not factorise easily)
* did not follow instructions that directed the student to use a particular method such as ‘implicit differentiation’ or to use a previous result, either by instructing students to use specified parts of the question or using the word ‘hence’
* did not read questions carefully and managed their time poorly, leaving little time to complete other questions
* lacked the appropriate detail; where several marks have been allocated, all relevant conceptual steps are required
* did not communicate a good knowledge of the algorithms covered by the course, often evident through incorrect application of techniques to solve questions
* seemed unfamiliar with the capability of their graphics calculator

Assessment Type 2: Mathematical Investigation (20%)

Students complete one mathematical investigation.

The subject of the investigation may be derived from one or more subtopics and should have *minimal* teacher direction. The task must afford students the opportunity to extend the investigation in an open-ended context. Students are encouraged to use a variety of mathematical (and other) software to enhance their investigation. It must be completed in a report format and must be no longer than 15 single-sided A4 pages with a minimum font size of 10. Appendices may be used to support the report, but are not part of the assessment decision unless they are part of the 15 pages. Teachers should provide feedback, where appropriate, on the suitability of the direction a student may take with their investigation, especially where the investigation topic was chosen by the student and provide feedback on one draft. In draft feedback, the teacher may direct the student’s attention to errors but must not explicitly correct these for the student.

Students provide evidence of their learning in relation to the following assessment design criteria:

* concepts and techniques
* reasoning and communication.

Teachers can elicit more successful responses by:

* ensuring that the format of the investigation allows for an open-ended exploration of a problem where the student can show the development and application of mathematical models through individual choices, refinements/improvements with justification for their rationale
* providing clear annotation on the student work by the teacher to assist the moderator in confirming the school decision
* providing examples in the task sheet of what could be modelled, and structuring the investigation to encourage students to focus on different models, extend their interests and explore more complex models
* ensuring that the investigation is at an appropriate level of complexity, aligns well with the Subject Outline and does not limit the student’s ability to achieve at the highest level
* not using question-and-answer style investigations, which limit student success
* ensuring the design of the task allows for and explicitly encourages the discussion of limitations and reasonableness of the modelling process. See points below.
	+ Tasks that are designed to look at the generation of curves or shapes by altering values within formulae is not likely to result in individual work that is sufficiently open-ended or allow for deep discussions concerning the reasonableness of solutions or limitations encountered.
	+ Examples of types of investigations that may limit student success are Bezier curves with a conjecture and proof that is not unique to the student, graphs of various rational functions, or graphs of other relations that may give altering shapes depending on values chosen. Some Bezier curve investigations may not reach a complex level of modelling and may limit the discussion of reasonableness and limitations.
	+ The most recently updated wine glass investigation on the SACE website allows an open-ended approach after initially being directed. Students need to execute a significantly open-ended section to produce an investigation at a complex level. For example, the modelling of pathways with parametric curves provides no direction and allows students to develop their own modelling, and as such, is an excellent exemplar.
* encouraging the correct use of notation and labelling of graphs, axes, scales, etc.
* assisting students with unfamiliar software so that they can represent graphs, etc. with appropriate information attached and using correct mathematical notation
* providing feedback through drafting and/or discussing the direction taken to ensure that what students plan to do will provide them with the opportunity to achieve at the higher-grade bands and the teacher may direct the student’s attention to errors but must not explicitly correct these for the student
* explaining clearly the 15-page (single-sided) limit and the corresponding appropriate use of appendices. For example, important or initial mathematical calculations should be provided in the main body of the report; however, repetitions of a calculation with variations to the figures can be provided in the appendices with the results clearly provided in the main body of the investigation (within a table or using some other concise manner of presentation)
* not using investigations that have published solutions such as those provided by MASA to ensure that student work is unique and authentic. Examples of several such investigations not to use include the Tennis application, and De Moivre’s theorem application, both which present as question-and-answer style.

The more successful responses commonly:

* provided detailed information about the investigation and the context in the real world in their introduction
* wrote in a report-style with clearly communicated processes linking their ideas and made use of the full 15-page limit
* were student-driven, and included both mathematical calculations and the use of technology with a focus on interpretation and evaluation of models that had been developed and applied in the chosen context
* read as a complete report, with sentences of explanation, not a series of dot-point-like ‘answers’ to an ‘assignment’
* included detailed explanations of all algebra, choices of values, and graphical work produced
* included appropriately-labelled graphical representations to enhance the discussion within the investigation
* successfully developed a modelling situation with clear explanation of the decisions made throughout the mathematical investigation justified with reference to the real-life context and/or cited research and referenced as appropriate. This included mathematical calculations for each stage of development of the model that were commensurate with the cognitive demands of Stage 2 Specialist Mathematics
* demonstrated understanding of the reasonableness of the mathematical results and the limitations of the modelling process used, with attempts to improve, expand on, and develop based on these reflections, as appropriate
* used appropriate mathematical software to enhance the quality of the investigation
* used mathematical notation, representations, and terminology appropriately and accurately
* effectively communicated mathematical ideas and reasoning to develop logical arguments
* used sub-headings throughout the investigation, which led the assessor through each stage of development and were more comprehensive than a series of paragraphs, calculations, and graphical representations
* formatted their document so the mathematical notation flowed properly, and headings didn’t appear at the bottom of one page and the content at the top of the next page
* used appendices appropriately for repeated algebraic calculations to arrive at results.

The less successful responses commonly:

* had a limited introduction to the investigation, giving the reader little insight into the nature of the problem and the investigation to be undertaken
* had limited supporting evidence of how the models were derived e.g. trial and error, Geogebra, researched and adapted
* provided little evidence of effective use of technology. The investigation is an ideal assessment to implement a range of technologies to represent and solve problems which leads to the development of the model
* read like a series of dot-point-answers, as if the student just listed responses to an assignment or worksheet
* did not provide explanations or reasoning for the decisions made throughout the investigation with little discussion around the reasonableness of results and limitations
* made poor use of notation and often did not fully identify graphs
* included little or no labelling of diagrams
* followed the early direction given, but did not achieve much more, often failing to attempt the open-ended part of the investigation or sometimes spending too much time on the directed part and too little on the open-ended part
* often presented well under the 15-page allowance, thereby limiting the depth of discussion possible
* appeared to not have submitted their draft to the teacher for feedback.

Operational Advice

Students should not speed-up the recording of their videos excessively in an attempt to condense more content into the maximum time limit.

If a video is flagged by moderators as impacted by speed, schools will be requested to provide a transcript and moderators will be advised to moderate based on the evidence in the transcript, only considering evidence up to the maximum word limit.

If the speed of the recording makes the speech incomprehensible, it affects the accuracy of transcriptions and it also impacts the ability of moderators to find evidence of student achievement against the performance standards.

External Assessment

Assessment Type 4: Examination

General

The examination consisted of two booklets. Booklet one was worth 55 marks and booklet two had longer questions with a total of 45 marks. As in past years, the cohort who undertook the examination was made up of those students who knew their work and produced good to very good results, but a proportion of students struggled to respond successfully.

Students found booklet one questions more accessible and many successfully displayed their knowledge. Questions within booklet two were found to be more difficult, but there were still many students with excellent work in both booklets.

General comments worth stressing:

* the ‘Show that …’ style of questions require students to show full working, displaying all steps of logic, for maximum marks. The style of solution here should be approaching one side of the given information and working towards developing the other side. The two sides should not be used together
* an ‘exact answer’ means the answer should be in rational or irrational form without approximations to decimal values
* students need to be reminded that if the answer is stated in the question, marks are awarded for providing the working steps needed to reach this answer
* knowledge of, and the use of, a graphics calculator is assumed
* poor notation was seen in student responses. Two areas of concern are the poor use of vector notation and integration notation
* students should also be mindful of using the variables in the question. For instance, if a function of  is stated, then student responses should be in terms of  not 
* students should recognise that earlier parts of a question are often relevant to the later parts of a longer question. Some questions, for instance, may state ‘hence’ or ‘using part (a)(i)’ to instruct students to follow on from previous work
* students should be aware of algebraic language. Some students did not use the brackets required to show a logical flow of their algebraic reasoning. This leads to errors in their mathematics
* students must set out mathematical induction proof appropriately to gain full marks
* students should ensure they answer a question on extra pages in the correct booklet. It is advisable that students indicate in the space for an answer if they are also using the extra page for more working. For example, ‘see page x’. The work on the extra pages must be labelled clearly.

Examination markers aim to award marks for evidence of student understanding in responding to examination questions wherever possible; however, students should be advised not to cross out their responses or attempted responses to questions in the examination booklet unless they are confident that no part of their response should be considered by the marker.

If a student crosses out a response and then decides that it was the correct (or the most correct) answer, then the student should indicate clearly to the marker which part of their response should be considered. This could be done by circling or highlighting all or part of the response that the student wants to be considered and write, for example, ‘please mark this work’. Students do not need to rewrite their answers in this case unless the crossing out has rendered the response unreadable.

Specific comments for the questions within booklets 1 and 2 are below.

Booklet 1

Question 1

A good start for most students.

(a) Very well done overall but students need to ‘show’ that once they establish that t = -1, the point resulting is (0, 4, 4) by substituting the t value into the given equation of the line.

(b) Mostly well done, with students finding the equation of the plane using the normal vector. Some attempted incorrectly to find a line.

Question 2

A good result for many students.

(a) Many achieved well but a reminder of the wording ‘exact’ requires an answer that is not approximated.

(b) (i) Well done by the majority.

 (ii) Many relied on their notes sheet for the derivative of arctan(ax), but the ‘Show that’ requirement meant that steps of logic were needed. That is, $\frac{dθ}{dt}=\frac{50}{50^{2}+x^{2}} \frac{dx}{dt}$ was the least requirement.

(c) The most successful students found  when . Finding the final answer to three significant figures was an issue for some students.

Question 3

Another question that was well done by the majority of students.

(a) (i) and (ii) Some students used the calculator efficiently, but some still take a longer approach for just the one mark.

(b) (i) Some careless work finding  instead of  .

(ii) A few stated a negative angle instead of  .

(c) Most students calculated the modulus correctly, but some struggled with finding the argument correctly.

Question 4

Students who set up the proof correctly and set their work out logically were the most successful. Many students fared well in this question.

(a) A reminder that it is necessary to state the proposition P(n) at the beginning of the proof if finding P(1), assuming P(k) and considering P(k+1). Some students had difficulty showing that when P(k) is true implying P(k+1) true. Also, it is worth noting that students who use the left-hand side and the desired result of the right-hand side of P(k+1) do not present a valid proof.

(b) Some good reasoning was seen, although some did have difficulty seeing how to use the result of part (a).

Question 5

Students who clearly showed row operations to the final required matrix were the most successful.

1. (i) Most students wrote the augmented matrix correctly, with only some including a variable instead of just its coefficient.

(ii) Clearly defined row operations were required. Some students did not show final steps towards the given result of the matrix. Working must be shown.

(b) Recognising that no solution is the representation in the diagram was missed by some students.

(c) Some were unsure of the interpretation required.

(d) (i) and (ii) Most found a=-1 and went on to find the equation of the line.

Question 6

This question was very well done by the majority of students.

(a) Many recognised the style of question and answered successfully, but some used different symbols.

(ii) Many recognised the separation of variables techniques, with only some omitting moduli and some not setting out their working with appropriate reasoning to obtain the given solution.

(b) Well done by most students. The most successful approaches took care with the asymptotic behaviour, the slope field lines and shape of a logistic curve.

(c) (i) (ii) (iii) Most correctly stated K=960. Part (ii), to a small degree, and part (iii) were best approached by using the calculator for ease and accuracy.

Question 7

This question was approached successfully by the majority of students. Throughout this question students need to be mindful of showing all the working when answers are given.

(a) Different approaches were possible, and the most successful students were careful with constants of integration.

(b) (i) (ii) Both parts were generally well done, but care must be taken when drawing graphs to clearly show intercepts and shape.

(c) (i) If working was clearly shown, students were successful.

(ii) Some students struggled with their working to obtain the required result. The most successful students used appropriate earlier work.

Many questions in booklet 1 were answered successfully by students.

Booklet 2

Question 8

This question was the most successfully approached problem.

(a) The most successful approach was to use the sine ratio.

(b) Well done but some students found  instead of .

(c) (i) Well done but some did not find the length of the cross product.

(ii) and (iii) both well done.

(iv) Many found t=2 but working must be shown for the resulting point of intersection.

(d) The most challenging part of the question. Similar triangles or the intersection of lines were the most popular approaches.

Question 9

Many students performed poorly in this question. The most successful students understood complex number skills well.

(a) Recognising that this question was asking to solve  was not apparent to some students.

(b) (i) (ii) The first part of this question was very poorly done with many not realising the triangle inequality was in play. The second part of the question was more successfully answered.

(c) (i) Poor algebra was seen far too frequently in student responses. The form  was sometimes written incorrectly as , meaning subsequent working was often incorrect, or not logical. Students must use logical reasoning with correct algebraic expressions.

(ii) Mixed responses were seen here, with z = 1 being correctly omitted.

(d) (i) and (ii) were poorly attempted, with many not showing the expansion of  and the negative angle versions to display the cancellation of the imaginary parts. If students recognised the double angle form , part (ii) was answered successfully.

(iii) Good use of the calculator saw students answer this question successfully, although being aware that the minimum value was required and not a point, is important.

(iv) Very few students answered this part correctly. The values of radians were required to be shown on the diagram.

Question 10

Some very good responses were seen for this question.

(a) The ‘Show that’ requirement in this question meant students needed to show  becomes . Many students did not supply enough working.

(b) Many drew good graphs and used the graphics calculator efficiently. The most successful students drew with care, plotting points to assist accuracy and ensuring the domain for *t* was correct. Students are reminded to use the scales given on the axes in the diagram.

(c) (i) Most students were successful in this question, but it is concerning that some students were not aware the velocity vector is of the form  .

(ii) Proved difficult for most students. Some did not expand the initial expression for speed correctly, and commonly the perfect square was not noticed by many.

(iii) Many started this question well, recognising the definite integral to be utilised. Some found the evaluation difficult and could not show the required result.

Overall, this exam saw some very good work from students, with some displaying excellent understanding of the concepts within the course. It is recommended that:

* students are advised to show all working when undertaking ‘Show that’ style problems.
* students read the front of the booklets and notice that spare pages within a booklet are to be used for questions within the same booklet.
* students use correct notation, algebra, and terminology in their work.