South Australian Certificate of Education

The purpose of this sample paper is to show the structure of the 130-minute Mathematical Methods examination and the style of questions that might be used. The examination will consist of questions that assess a *selection* of the key questions and key concepts from across the six topics.

1

Mathematical Methods
November 2020 sample paper

Question booklet 1

- Questions 1 to 6 (52 marks)
- Answer ***all*** questions
- Write your answers in this question booklet
- You may write on page 16 if you need more space
- Allow approximately 65 minutes
- Approved calculators may be used — complete the box below

Examination information

Materials
- Question booklet 1
- Question booklet 2
- Formula sheet
- SACE registration number label

Instructions
- Show appropriate working and steps of logic in the question booklets
- State all answers correct to three significant figures, unless otherwise instructed
- Use black or blue pen
- You may use a sharp dark pencil for diagrams and graphical representations

Total time: 130 minutes
Total marks: 96

Attach your SACE registration number label here	**Graphics calculator** 1. Brand _____ Model _____ 2. Brand _____ Model _____

Government of South Australia

Question 1 (8 marks)

Find $\frac{dy}{dx}$ for each of the following. There is no need to simplify your answers.

(a) $y = x^2 e^x$.

(2 marks)

(b) $y = (x + \cos x)^4$.

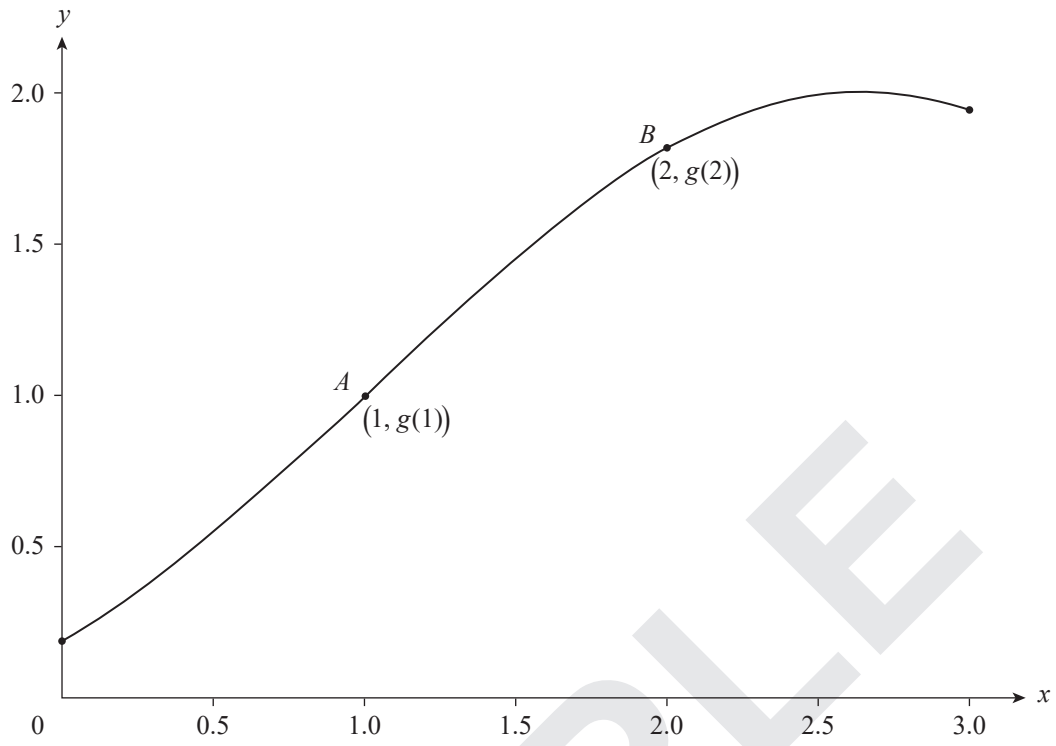
(3 marks)

(c) $y = \dfrac{\ln\left(x^2 + 3x\right)}{x}$.

(3 marks)

Question 2 (4 marks)

The graph of the curve $y = g(x)$ for $0 \leq x \leq 3$ is shown below.



Points A and B lie on the curve. Point A is the only point of inflection.

(a) Which *one* of the following statements is true? Tick the appropriate box, and justify your answer.

$g'(1) < g'(2)$ ☐

$g'(1) = g'(2)$ ☐

$g'(1) > g'(2)$ ☐

Justification:

(2 marks)

(b) Which *one* of the following statements is true? Tick the appropriate box, and justify your answer.

$g''(2) < 0$ ☐

$g''(2) = 0$ ☐

$g''(2) > 0$ ☐

Justification:

(2 marks)

Question 3 (6 marks)

The owner of a local pet food shop is offering an incentive program to attract customers.

The shop owner has placed 50 balls in a bucket. Each ball is labelled with a percentage discount that the customer will be given after their purchase total is tallied. The customer draws a ball randomly from the bucket, and the percentage discount on the ball is applied to the purchase total. The ball is then placed back in the bucket before the next customer draws a ball.

Of the 50 balls in the bucket:

- one ball is labelled '100% discount'
- two balls are labelled '50% discount'
- four balls are labelled '25% discount'
- eight balls are labelled '20% discount'.

The remaining 35 balls are labelled '10% discount'.

(a) What is the probability that a customer will draw the ball labelled '100% discount'?

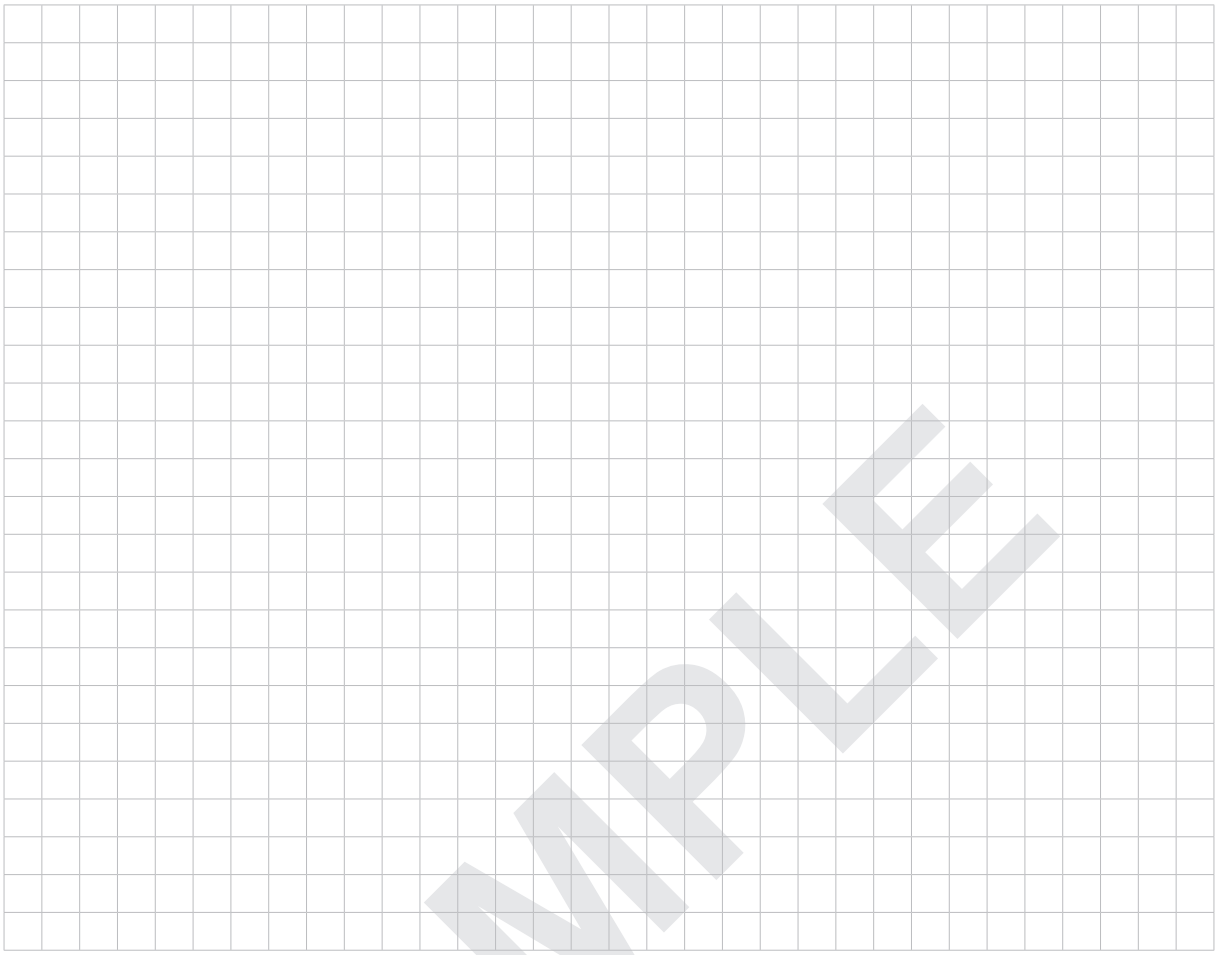
(1 mark)

(b) If 500 customers each draw one ball from the bucket, what is the probability that more than 10 of these customers will draw the ball labelled '100% discount'?

(2 marks)

(c) The shop owner could offer an alternative incentive program, in which a 15% discount is given to all customers.

In the long run, which of these two incentive schemes will cost the shop owner more? Explain your answer.



(3 marks)

Question 4 (12 marks)

Consider the function $f(x) = \ln(3x + 8)$.

(a) For what values of x is $f(x)$ undefined?

(1 mark)

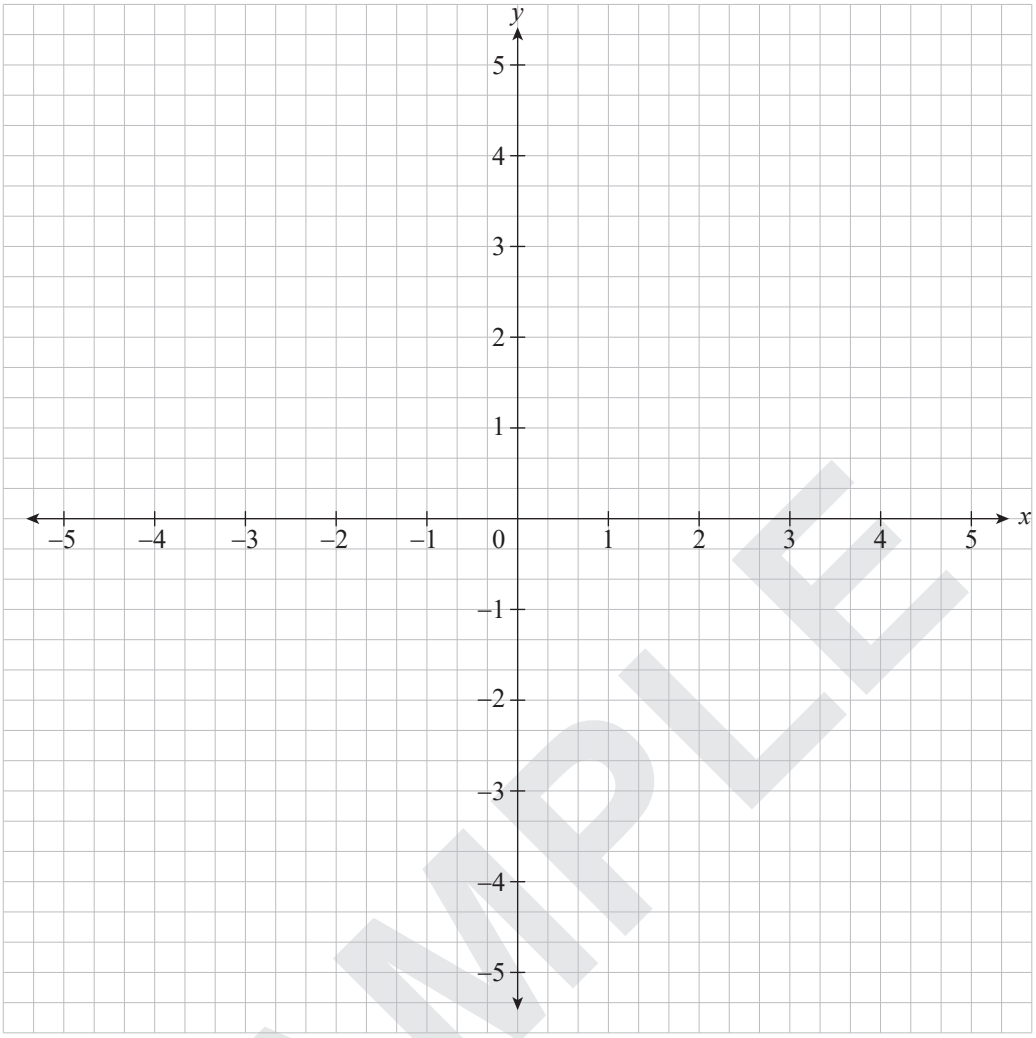
(b) Using algebra, solve the equation $f(x) = 0$.

(2 marks)

(c) Evaluate $f(0)$.

(1 mark)

(d) On the axes below, sketch the graph of $y = f(x)$, clearly showing and labelling the information found in parts (a), (b), and (c).



(3 marks)

Question 4 continues on page 10.

Now consider the function $g(x) = \ln(bx + c)$, where $b > 0$ and $c > 0$.

(e) Find the equation of the asymptote.

(2 marks)

(f) Find the coordinates of the x-intercept.

(1 mark)

(g) Find the slope of the tangent to $g(x)$ at $x = 0$.

(2 marks)

Question 5 (8 marks)

Source: Adapted from
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In one particular city, there have been many thefts of student possessions from schools. In an attempt to reduce these thefts, the government has provided new, lockable cupboards to all schools in this city. Each student at each school has been assigned a separate cupboard, and has been instructed to store their possessions in their cupboard and always keep it locked.

The government conducts a survey of school students, to determine how many students always keep their cupboard locked.

From a sample of 320 junior secondary students, the following 95% confidence interval for the proportion of junior secondary students who always keep their cupboard locked is calculated:

$$0.797 \leq p \leq 0.878.$$

(a) Explain the meaning of this confidence interval.

(2 marks)

(b) (i) What proportion of the students in this sample always keep their cupboard locked?

(1 mark)

(ii) Hence how many of the 320 students in this sample always keep their cupboard locked?

(1 mark)

(c) It is suggested that senior secondary students are less likely than junior secondary students to always keep their cupboard locked. Therefore, the government surveys a sample of 120 senior secondary students, and finds that 74 of these students always keep their cupboard locked.

(i) Calculate a 95% confidence interval for the proportion of senior secondary students who always keep their cupboard locked.

(2 marks)

(ii) Can the government conclude that senior secondary students are less likely than junior secondary students to always keep their cupboard locked? Explain your answer.

(2 marks)

Question 6 (14 marks)

(a) (i) Find $\frac{dy}{dx}$ if $y = xe^{-x}$.

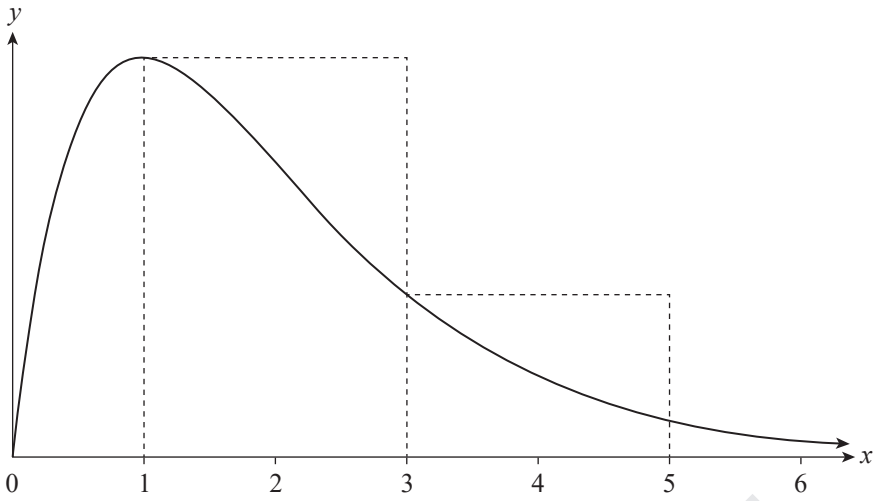
(2 marks)

(ii) Hence show that $\int xe^{-x}dx = -xe^{-x} - e^{-x} + c$.

(3 marks)

Question 6 continues on page 14.

The graph of $y = f(x)$, where $f(x) = xe^{-x}$, is shown below.



(b) An estimate is required for the area bounded by $f(x)$, the x-axis, and the vertical lines $x = 1$ and $x = 5$.

(i) Two rectangles, each 2 units wide, have been added to the graph to be used in the calculation of an overestimate for this area.

Calculate this overestimate by finding the sum (S) of the areas of these two rectangles, correct to three decimal places.

(2 marks)

(ii) A new overestimate of the same area can be calculated, using *four* rectangles of equal width.

(1) On the graph above, draw four rectangles that could be used to calculate a new overestimate. (1 mark)

(2) Calculate this new overestimate, giving your answer correct to three decimal places.

(2 marks)

(c) With reference to part (a)(ii), find the area bounded by $f(x)$, the x-axis, and the vertical lines $x = 1$ and $x = 5$.

(3 marks)

(d) Refer to your overestimate calculations from part (b) and your answer to part (c).

Comment on the effect that increasing the number of rectangles used in your calculations has on the accuracy of the area estimates that you obtained.

(1 mark)

You may write on this page if you need more space to finish your answers to any of the questions in Question booklet 1. Make sure to label each answer carefully (e.g. 5(b)(ii) continued).