**STAGE 2 MATHEMATICAL METHODS**

**PROGRAM 1**

This program is for a cohort of students studying Stage 2 Mathematical Methods. It is assumed that students have completed Topics 1-6 from Stage 1 Mathematics.

**Topic 1 – Further Differentiation and Applications (10 Weeks)**

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| **Term**  **week** | **Subtopic** | **Concepts and Content**  Technology is incorporated into all aspects of this topic as appropriate | **Assessment Task** |
| 1-1 | 1.1  Introductory Differential Calculus | Derivative of polynomial functions and power functions  Displacement and velocity  Applications to modelling  Local maxima and minima  Increasing and decreasing functions |  |
| 1-2 | 1.2  Differentiation Rules | Differentiation rules   * Chain rule * Product rule * Quotient rule |  |
| 1-3 | 1.3  Exponential Functions | The derivative of and y  Modelling growth and decay using exponential functions including the surge function and the logistic function. |  |
| 1-4 | Using exponential functions   * Slope of tangents to graphs of functions * Local maxima and minima * Increasing and decreasing functions * Displacement and velocity |  |
| 1-5 | Applications to model actual scenarios using exponential functions, including those with growth and decay. | **SAT 1 – Differential Calculus (1.1-1.3)**  **Part 1 – No calculator**  **Part 2 – Calculator permitted** |
| 1-6 | 1.4  Trigonometric Functions | Revision of graphing trigonometric functions including radian measure.  Derivatives of sine, cosine and tan.  Use of differentiation rules with trigonometric functions. |  |
| 1-7 | Using derivatives of trigonometric functions   * Slope of tangents to graphs of functions * Local maxima and minima * Increasing and decreasing functions * Displacement and velocity |  |
| 1-8 | Modelling periodic scenarios such as tidal heights, temperature changes and AC voltages. |  |
| 1-9 | 1.5  The Second Derivative | The notation  The relationship between the function, its derivative, and the second derivative from graphical examples.  Curve sketching including concavity and points of inflection. |  |
| 1-10 | Applications to acceleration and increasing and decreasing velocity from the displacement function. | **SAT 2 – Differential Calculus (1.4-1.5)** |

**Topic 2 – Discrete Random Variables (4 weeks)**

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| **Term**  **Week** | **Subtopic** | **Concepts and Content**  Technology is incorporated into all aspects of this topic as appropriate | **Assessment Task** |
| 2-1 | 2.1  Discrete Random Variables | The probability of each value of a random variable is constant.  Distinguish between continuous and discrete random variables.  The probability function and its properties.  Displaying the probability distribution.  Uniform and non-uniform discrete random variables. | **INVESTIGATION**  Designing a rollercoaster |
| 2-2 | Estimating probabilities of discrete random variables.  Expected value and its purpose in estimating the centre of the distribution and the sample mean.  Standard deviation and its purpose in measurement of the spread of the distribution. |  |
| 2-3 | 2.2  The Bernoulli Distribution | Bernoulli variables are discrete random variables with only two outcomes.  The Bernoulli distribution  The mean and standard deviation. |
| 2-4 | 2.3  Repeated Bernoulli Trials and the Binomial Distribution | The binomial random variable and the binomial distribution.  The mean and the standard deviation  Modelling scenarios using the binomial distribution.  Finding binomial probabilities using  and electronic technology.  The shape of the binomial distribution for large values of . | **SAT 3 – Discrete Random Variables (2.1-2.3)** |

**Topic 3 – Integral Calculus (6 weeks)**

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| Term  week | **Subtopic** | **Concepts and Content**  Technology is incorporated into all aspects of this topic as appropriate | **Assessment Task** |
| 2-5 | 3.1  Anti-differentiation | Changing a derivative to the original function.  The indefinite integral ; and  Integrals of including consideration of functions of the form ) |  |
| 2-6 | Using  Determining the specific constant of integration. |  |
| 2-7 | 3.2  The Area under Curves | Estimating the area under a curve using the sums of upper and lower rectangles of equal width.  Strategies to improve the estimate.  Use of electronic technology to find the area.  Using the terminology for the exact area for a positive continuous function, − for a negative continuous function, and for the area between two curves where is above .  The observations  = 0 and |  |
| 2-8 | 3.3  Fundamental Theorem of Calculus | ; and hence the verification of  = 0 and .  Evaluating the exact area under a curve and the area between two curves. |  |
| 2-9 | 3.4  Applications of Integration | The area of cross-sections.  The total change in a quantity given the rate of change over a time period. |  |
| 2-10 | The distance travelled and position from a velocity function.  The velocity from an acceleration function. | **SAT 4 – Integral Calculus (3.1-3.4)**  **Part 1 – No calculator**  **Part 2 – Calculator permitted** |

**Topic 4 – Logarithmic Functions (3 weeks)**

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| **Term**  **week** | **Subtopic** | **Concepts and Content**  Technology is incorporated into all aspects of this topic as appropriate | **Assessment Task** |
| 3-1 | 4.1  Using Logarithms for Solving Exponential Equations | Given then leading to  when then  Solving exponential equations and the revision of the log laws.  Using log scales to linearize an exponential scale. |  |
| 3-2 | 4.2  Logarithmic Functions and their Graphs | The graph of and its properties.  The graph of functions in the form  The relationship between the graphs of and |  |
| 3-3 | 4.3  Calculus of Logarithmic Functions | The derivatives of and  provided is positive.  Problem solving using the derivatives of logarithmic functions.  Applications of logarithmic functions. | **SAT 5 – Logarithmic Functions (4.1-4.3)** |

**Topic 5 – Continuous Random Variables and the Normal Distribution (4 weeks)**

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| Term  week | **Subtopic** | **Concepts and Content**  Technology is incorporated into all aspects of this topic as appropriate | **Assessment Task** |
| 3-4 | 5.1  Continuous Random Variables | Comparing discrete and continuous random variables.  The probability of a specific range of values.  Probability density functions and their graphs.  The mean from and the  standard deviation |  |
| 3-5 | 5.2  Normal Distributions | Conditions for a normal random variable.  The key properties of normal distributions.  The probability density function  Using electronic technology to calculate proportions, probabilities, and the upper or lower limit of a certain proportion.  The standard normal distribution with and  Standardising a normal distribution using . |  |
| 3-6 | 5.3  Sampling | Sampling Distributions  the outcome of adding n independent observations of X.  the outcome of averaging n independent observations of X.  If then and provided is sufficiently large.\* |  |
| 3-7 | Simple random sample  If then for a sample size  Central limit theorem. |  |

\* using the notation for a normal distribution with mean and standard deviation

**Topic 6 – Sampling and Confidence Intervals (3 weeks)**

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| Term  week | **Subtopic** | **Concepts and Content**  Technology is incorporated into all aspects of this topic as appropriate | **Assessment Task** |
| 3-8 | 6.1  Confidence Intervals for a Population Mean | Sample means are continuous random variables.  Distribution of sample means will be approximately normal for a sufficiently large sample.  A confidence interval can be created around the sample mean that may contain the population mean. If is the sample mean then the confidence interval is , where is determined by the confidence level that the interval will contain the population mean. |  |
| 3-9 | 6.2  Population Proportions | Concept of a population proportion .  Sample proportion is a discrete random variable with a mean and standard deviation .  As the sample size increases the distribution of becomes more like a normal distibution. |  |
| 3-10 | 6.3  Confidence Intervals for a Population Proportion | A confidence interval can be created around the sample proportion that may contain the population proportion.  If is the sample mean then the confidence interval is , where is determined by the confidence that the interval will contain the population mean. | **SAT 6 – Statistics (5.1-5.3 and 6.1-6.3)** |

**Revision**

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| Term  week | **Subtopic** | **Concepts and Content** | **Assessment Task** |
| 4-1 |  | Revision |  |
| 4-2 |  | Revision |  |
| 4-3 |  | Swot Vac |  |
| 4-4 |  | Exam |  |

**Notes**Please note that this is a working document and may be adjusted as the course progresses.

Suggested allocation of time for this program:

Topic 1: Further Differentiation and Applications (10 weeks)  
Topic 2: Discrete Random Variables (4 weeks)  
Topic 3: Integral Calculus (6 weeks)   
Topic 4: The logarithmic function (3 weeks)  
Topic 5: Continuous Random Variables and the Normal Distribution (4 weeks)  
Topic 6: Sampling and Confidence Intervals (3 weeks)

Assessment consists of three assessment types:

School-based Assessment (70%)

* Assessment Type 1: Skills and Applications Tasks (50%)
* Assessment Type 2: Mathematical Investigation (20%)

External Assessment (30%)

* Assessment Type 3: Examination (30%).