**QUESTION 1** **(2 marks)**

A police "identi-kit" box for men contains:

| | |
|---|---|
| 6 different face shapes | 8 different hair lines |
| 4 different eyebrow styles | 10 different eye shapes |
| 10 different noses | 5 different moustaches |
| 8 different sets of lips | 5 different beards |

(a) How many different identi-kit pictures can be made up?

(1 mark)

(b) If a suspect is masked from the nose downward (obscuring his nose, moustache, lips, beard and face shape), how many different identi-kit pictures of a masked suspect can be made up?

(1 mark)

**QUESTION 2** **(2 marks)**

(a) Express in factorial form $\frac{9 \times 8 \times 7}{3 \times 2 \times 1}$.

(1 mark)

(b) Simplify $\frac{(n+2)!}{(n-1)!}$

(1 mark)

**QUESTION 3** **(5 marks)**

A tyre manufacturer claims that under normal driving conditions for a particular type of tyre the legal tread life ($X$), measured in kilometres travelled, follows a normal distribution with a mean $\mu$ = 60 000 km and a standard deviation $\sigma$ = 5000 km.

(a) Find the probability that a tyre will have a tread life of less than 58 000 km

(1 mark)

(b) Find the probability that a tyre will have a tread life between 50 000 km and 65,000 km.

(1 mark)

(c) Find the probability that a tyre will have a tread life of more than 63 000 km.

(1 mark)

(d) The manufacturer sells *one million* of these tyres and offers a warranty that they will last at least 45 000 km. Approximately how many tyres will wear out before the warranty expires?

(2 marks)

**QUESTION 4** **(8 marks)**

(a) Use the Binomial Expansion to expand and simplify $(1-2x)^5$

(3 marks)

(b) Find the 4th term in the expansion of $\left(\frac{1}{x}-2\right)^{10}$ but do not simplify your answer.

(2 marks)

(c) In the expansion of $(2-x)^7(3x-1)$, find the coefficient of $x^3$.

(3 marks)

**QUESTION 5** **(4 marks)**

The weights of crabs from a crab farm are found to be normally distributed with mean 3 kg and standard deviation 0.5 kg. The crabs fall into three categories:

| | Discarded | Public | Gourmet |
|---|---|---|---|
| **Weight** | less than 2.5 kg | between 2.5 kg & 3.5 kg | greater than 3.5 kg |

(a) Find the probability that a crab caught from the farm will be labelled

(i) as a "Discarded"

(1 mark)

(ii) as a "Gourmet"

(1 mark)

(b) Restaurants are not satisfied with the Gourmet label. They require that a crab's weight be in the top 5% of weights from the crab farm in order to be labelled as Gourmet. What is the minimum weight for these crabs?

(2 marks)

**QUESTION 6** **(4 marks)**

(a) Z is normally distributed with mean $\mu = 0$ and standard deviation $\sigma = 1$ .

Determine the value of *k* for which $P(Z < k) = 0.85$.

(1 mark)

(b) The times taken for students to complete this Mathematics test are normally distributed with a standard deviation of 10 minutes.

If 85% of people took less than 50 minutes to complete the test, find the mean time taken to complete the test.

(3 marks)

**QUESTION 7 (8 marks)**

A computer is programmed to list all of the permutations of the 8 letters of the word COMPUTER.

(a) How many permutations are there?

(1 mark)

How many of these permutations:

(b) start with C?

(1 mark)

(c) start with a consonant?

(1 mark)

(d) start and end with a consonant?

(1 mark)

(e) end with the letters ER in that order?

(1 mark)

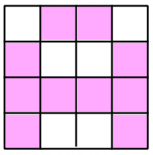
(f) end with the letters E and R in either order?

(1 mark)

(g) have the letters E and R together?

(2 marks)

**QUESTION 8 (5 marks)**



A part of a computer screen is divided into 16 square regions.

The programmer wishes to create patterns by shading in squares.

How many different patterns are possible if:

(a) ten squares are to be shaded? (One such pattern is shown above)

(1 mark)

(b) eight squares are to be shaded?

(1 mark)

(c) eight squares to be shaded but must include the corners?

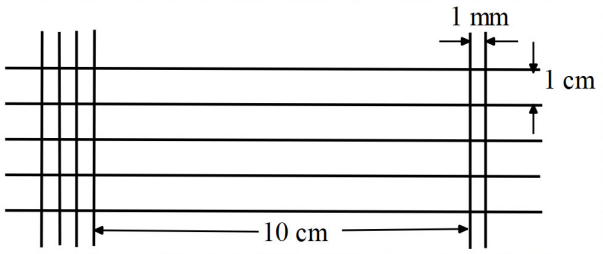
(2 marks)

(d) all 16 squares are to be shaded?

(1 mark)

**QUESTION 9** (2 marks)

There are 5 horizontal lines crossed by 6 vertical lines. The distances between the lines are indicated on the diagram.




How many different rectangles, with areas greater than 9 square centimetres, can be formed?

(2 marks)

**QUESTION 1** **(2 marks)**

A police "identi-kit" box for men contains:

| | |
|---|---|
| 6 different face shapes | 8 different hair lines |
| 4 different eyebrow styles | 10 different eye shapes |
| 10 different noses | 5 different moustaches |
| 8 different sets of lips | 5 different beards |

(a) How many different identi-kit pictures can be made up?

$6 \times 4 \times 10 \times 8 \times 8 \times 10 \times 5 \times 5 = 3{,}840{,}000$ ✓

(1 mark)

(b) If a suspect is masked from the nose downward (obscuring his nose, moustache, lips, beard and face shape), how many different identi-kit pictures of a masked suspect can be made up?

$8 \times 4 \times 10 = 320$ ✓

(1 mark)

**QUESTION 2** **(2 marks)**

(a) Express in factorial form $\frac{9 \times 8 \times 7}{3 \times 2 \times 1}$.

$\frac{9 \times 8 \times 7}{3 \times 2 \times 1} = \frac{9!}{3!6!}$ ✓

(1 mark)

(b) Simplify $\frac{(n+2)!}{(n-1)!} = (n+2)(n+1)n$ ✓

(1 mark)

**QUESTION 3** **(5 marks)**

A tyre manufacturer claims that under normal driving conditions for a particular type of tyre the legal tread life ($X$), measured in kilometres travelled, follows a normal distribution with a mean $\mu$ = 60 000 km and a standard deviation $\sigma$ = 5000 km.

(a) Find the probability that a tyre will have a tread life of less than 58 000 km

$P(X < 58000) \approx 0.345$ ✓

(1 mark)

(b) Find the probability that a tyre will have a tread life between 50 000 km and 65,000 km.

$P(50000 < X < 65000) \approx 0.819$ ✓

(1 mark)

(c) Find the probability that a tyre will have a tread life of more than 63 000 km.

$P(X > 63000) \approx 0.274$ ✓

(1 mark)

(d) The manufacturer sells *one million* of these tyres and offers a warranty that they will last at least 45 000 km. Approximately how many tyres will wear out before the warranty expires?

$P(X < 45000) \approx 0.00135$ ✓

$\therefore .00135 \times 1000000 \approx 1350$ tyres ✓

(2 marks)

**QUESTION 4** **(8 marks)**

(a) Use the Binomial Expansion to expand and simplify $(1-2x)^5$

$1 + 5(-2x) + 10(-2x)^2 + 10(-2x)^3 + 5(-2x)^4 + (-2x)^5$ ✓

$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$ ✓ ✓

(3 marks)

(b) Find the 4th term in the expansion of $\left(\frac{1}{x}-2\right)^{10}$ but do not simplify your answer.

$r = 3 \quad \binom{10}{3}\left(\frac{1}{x}\right)^7(-2)^3$ ✓ ✓

or $\quad 120 \times \frac{1}{x^7} \times -8 = -\frac{960}{x^7}$

(2 marks)

(d) In the expansion of $(2-x)^7(3x-1)$, find the coefficient of $x^3$.

$= 3x(2-x)^7 - 1(2-x)^7$

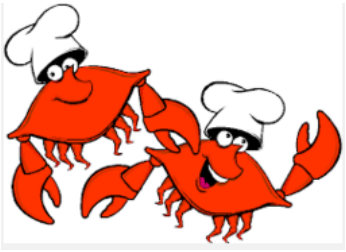
$\therefore x^3$: $3x\binom{7}{2}2^5 \times (-x)^2 - 1\binom{7}{3}2^4 \times (-x)^3$ ✓ ✓

$\therefore$ Coefficient of $x^3 = 3\binom{7}{2}2^5 + \binom{7}{3}2^4 = 2016 + 560 = 2576$ ✓

(3 marks)

**QUESTION 5** **(4 marks)**

The weights of crabs from a crab farm are found to be normally distributed with mean 3 kg and standard deviation 0.5 kg. The crabs fall into three categories:



| | **Discarded** | **Public** | **Gourmet** |
|---|---|---|---|
| Weight | less than 2.5 kg | between 2.5 kg & 3.5 kg | greater than 3.5 kg |

(a) Find the probability that a crab caught from the farm will be labelled

(i) as a "Discarded"

$p(x < 2.5) \approx 0.159$ ✓

(1 mark)

(ii) as a "Gourmet"

$P(X > 3.5) \approx 0.159$ ✓

(1 mark)

(b) Restaurants are not satisfied with the Gourmet label. They require that a crab's weight be in the top 5% of weights from the crab farm in order to be labelled as Gourmet. What is the minimum weight for these crabs?

$P(X > k) = 0.05 \Rightarrow k \approx 3.82$ ✓

$\therefore$ Minimum weight is $3.82\ kg$ ✓

(2 marks)

**QUESTION 6** **(4 marks)**

(a) Z is normally distributed with mean $\mu = 0$ and standard deviation $\sigma = 1$ .

Determine the value of *k* for which $P(Z < k) = 0.85$.

$P(Z < k) = 0.85 \Rightarrow k \approx 1.04$ ✓

(1 mark)

(b) The times taken for students to complete this Mathematics test are normally distributed with a standard deviation of 10 minutes.

If 85% of people took less than 50 minutes to complete the test, find the mean time taken to complete the test.

$P(X < 50) = 0.85 \Rightarrow P(Z < k) = 0.85$

$\therefore \frac{50 - \mu}{10} \approx 1.04$ ✓

$\therefore \mu \approx 50 - 1.04 \times 10 \approx 39.6$ ✓

Mean time to do test is 39.6 minutes

(3 marks)

**QUESTION 7** **(8 marks)**

A computer is programmed to list all of the permutations of the 8 letters of the word COMPUTER.

(a) How many permutations are there?

$8! = 40320$ ✓

(1 mark)

How many of these permutations:

(b) start with C?

$1 \times 7! = 5040$ ✓

(1 mark)

(c) start with a consonant?

$5 \times 7! = 25200$ ✓

Consonants are $C, M, P, T, R$

(1 mark)

(d) start and end with a consonant?

$5 \times 6! \times 4 = 14400$ ✓

(1 mark)

(e) end with the letters ER in that order?

$6! \times 1 \times 1 = 720$ ✓

(1 mark)

(f) end with the letters E and R in either order?

$6! \times 2 \times 1 = 1440$ ✓

(1 mark)

(g) have the letters E and R together?

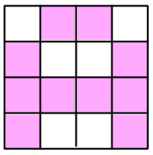
$ER\_\_\_\_\_\_\_$
$\_ER\_\_\_\_\_\_$
$\_\_ER\_\_\_\_\_$
$\_\_\_ER\_\_\_\_$
$\_\_\_\_ER\_\_\_$
$\_\_\_\_\_ER\_\_$
$\_\_\_\_\_ER\_$
$\_\_\_\_\_ER$

Letters can be other way around

$2 \times 7 \times 6!$ ✓ $= 10080$ ✓

(2 marks)

**QUESTION 8** **(5 marks)**



A part of a computer screen is divided into 16 square regions.

The programmer wishes to create patterns by shading in squares.

How many different patterns are possible if:

(a) ten squares are to be shaded? (One such pattern is shown above)

$\binom{16}{10} = 8008$ ✓ Mark for correct expression too

(1 mark)

(b) eight squares are to be shaded?

$\binom{16}{8} = 12870$ ✓

(1 mark)

(c) eight squares to be shaded but must include the corners?

$\binom{4}{4} \times \binom{12}{4} = 495$ ✓ ✓

(2 marks)

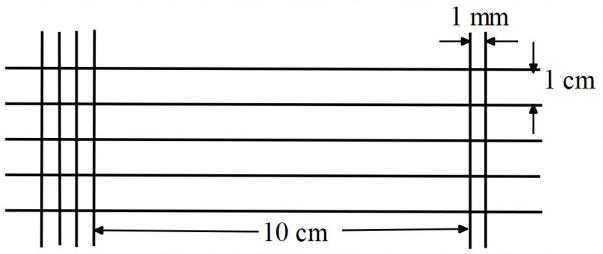
(d) all 16 squares are to be shaded?

$\binom{16}{16} = 1$ ✓

(1 mark)

**QUESTION 9** (2 marks)

There are 5 horizontal lines crossed by 6 vertical lines. The distances between the lines are indicated on the diagram.




How many different rectangles, with areas greater than 9 square centimetres, can be formed?

Vertical Sides $\binom{4}{1}$ and $\binom{2}{1}$

Horizontal Sides $\binom{5}{2}$

$\therefore$ Rectangles $= \binom{4}{1} \times \binom{2}{1} \times \binom{5}{2}$ ✓ $= 80$ ✓