# STAGE 1 MATHEMATICS

**Assessment Type 2: Mathematical Investigation**

### INVESTIGATING THE FEATURES OF POLYNOMIALS

### Task

A real polynomial function of degree 4 is a function of the form  where $a, b, c, d,$ and $e $ are real numbers, .

Every real polynomial of degree 4 can be factorised into one of seven forms. In your investigation you will consider a number of the seven forms.

The aim of this project is to investigate the graphs of real polynomial functions of degree 4 with particular reference to:

* the number of turning points
* the number of inflection points.

**Part A**

1. Investigate, using an appropriate graphing package, at least ***three*** real polynomials with four distinct real linear factors of the form.

Present the information and a small graph of each into a table. An example of the table you could use is presented below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Function** | **Roots used** | **Number of turning points** | **Number of points of inflection** | **Graph** |
|  |  |  |  |  |

1. Investigate at least ***three*** real polynomials with five distinct real linear factors of the form

 $p\left(x\right)=a\left(x-α\right)\left(x-β\right)\left(x-γ\right)\left(x-δ\right)\left(x-ε\right), a\ne 0$. Present the information in a table as above.

1. Present a possible conjecture in regard to the number of turning points and points of inflection for a polynomial of degree, $n$, with $n$ distinct real linear factors.
2. Support your conjecture by investigating another polynomial with a degree of your choice.

**Part B**

1. Investigate at least ***three*** real polynomials with a squared real linear factor and two distinct real linear factors,. Present the information in a table as above.
2. Extend this investigation to polynomials of degree five and beyond, with only one squared real linear factor.

$p\left(x\right)=a\left(x-α\right)^{2}\left(x-β\right)\left(x-γ\right)\left(x-δ\right), a\ne 0$.

1. Present a possible conjecture in regard to the number of turning points and points of inflection for a polynomial of this form.
2. Support your conjecture by investigating another polynomial with a degree of your choice.

**Part C**

There are several other possible combinations of factors for a quartic polynomial. Continue your investigation into at least two of these other possibilities, including a polynomial having a real quadratic factor with complex conjugate roots.

### Conclusion

You must write a conclusion summarising your findings.

The format of an investigation report may be written or multimodal.

The investigation report should be a **maximum of 8 pages** if written, or the equivalent in multimodal form.

Your report on the mathematical investigation should include the following:

* an outline of the problem and context
* the method required to find a solution, in terms of the mathematical model or strategy used
* the application of the mathematical model or strategy, including:
	+ relevant data and/or information
	+ mathematical calculations and results, using appropriate representations
	+ the analysis and interpretation of results, including consideration of the reasonableness and limitations of the results
* the results and conclusions in the context of the problem
* a bibliography and appendices, as appropriate.

**Assessment Design Criteria**

**Concepts and Techniques**

CT1 Knowledge and understanding of concepts and relationships

CT2 Selection and application of mathematical techniques and algorithms to find solutions to problems in a variety of contexts

CT3 Application of mathematical models

CT4 Use of electronic technology to find solutions to mathematical problems

**Reasoning and Communication**

RC1 Interpretation of mathematical results

RC2 Drawing conclusions from mathematical results, with an understanding of their reasonableness and limitations

RC3 Use of appropriate mathematical notation, representations, and terminology

RC4 Communication of mathematical ideas and reasoning to develop logical arguments

RC5 Development and testing of valid conjectures

Performance Standards for Stage 1 Mathematics

|  |  |  |
| --- | --- | --- |
|  | **Concepts and Techniques** | **Reasoning and Communication** |
| **A** | Comprehensive knowledge and understanding of concepts and relationships.Highly effective selection and application of mathematical techniques and algorithms to find efficient and accurate solutions to routine and complex problems in a variety of contexts. Successful development and application of mathematical models to find concise and accurate solutions.Appropriate and effective use of electronic technology to find accurate solutions to routine and complex problems. | Comprehensive interpretation of mathematical results in the context of the problem.Drawing logical conclusions from mathematical results, with a comprehensive understanding of their reasonableness and limitations.Proficient and accurate use of appropriate mathematical notation, representations, and terminology.Highly effective communication of mathematical ideas and reasoning to develop logical and concise arguments.Effective development and testing of valid conjectures. |
| **B** | Some depth of knowledge and understanding of concepts and relationships.Mostly effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine and some complex problems in a variety of contexts.Some development and successful application of mathematical models to find mostly accurate solutions.Mostly appropriate and effective use of electronic technology to find mostly accurate solutions to routine and some complex problems. | Mostly appropriate interpretation of mathematical results in the context of the problem.Drawing mostly logical conclusions from mathematical results, with some depth of understanding of their reasonableness and limitations.Mostly accurate use of appropriate mathematical notation, representations, and terminology.Mostly effective communication of mathematical ideas and reasoning to develop mostly logical arguments. Mostly effective development and testing of valid conjectures.  |
| **C** | Generally competent knowledge and understanding of concepts and relationships.Generally effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine problems in a variety of contexts.Successful application of mathematical models to find generally accurate solutions.Generally appropriate and effective use of electronic technology to find mostly accurate solutions to routine problems. | Generally appropriate interpretation of mathematical results in the context of the problem. Drawing some logical conclusions from mathematical results, with some understanding of their reasonableness and limitations. Generally appropriate use of mathematical notation, representations, and terminology, with reasonable accuracy.Generally effective communication of mathematical ideas and reasoning to develop some logical arguments. Development and testing of generally valid conjectures. |
| **D** | Basic knowledge and some understanding of concepts and relationships.Some selection and application of mathematical techniques and algorithms to find some accurate solutions to routine problems in some contexts.Some application of mathematical models to find some accurate or partially accurate solutions.Some appropriate use of electronic technology to find some accurate solutions to routine problems. | Some interpretation of mathematical results.Drawing some conclusions from mathematical results, with some awareness of their reasonableness or limitations.Some appropriate use of mathematical notation, representations, and terminology, with some accuracy.Some communication of mathematical ideas, with attempted reasoning and/or arguments.Attempted development or testing of a reasonable conjecture. |
| **E** | Limited knowledge or understanding of concepts and relationships.Attempted selection and limited application of mathematical techniques or algorithms, with limited accuracy in solving routine problems.Attempted application of mathematical models, with limited accuracy.Attempted use of electronic technology, with limited accuracy in solving routine problems. | Limited interpretation of mathematical results.Limited understanding of the meaning of mathematical results, their reasonableness or limitations.Limited use of appropriate mathematical notation, representations, or terminology, with limited accuracy.Attempted communication of mathematical ideas, with limited reasoning.Limited attempt to develop or test a conjecture. |