2020 Mathematical Methods Subject Assessment Advice

Overview

Subject assessment advice, based on the previous year’s assessment cycle, gives an overview of how students performed in their school and external assessments in relation to the learning requirements, assessment design criteria, and performance standards set out in the relevant subject outline. They provide information and advice regarding the assessment types, the application of the performance standards in school and external assessments, and the quality of student performance.

In 2020 the assessment specifications had temporary changes introduced to reduce assessment demands due to COVID-19. The flexibilities in the School Assessment component were approved by the COVID-19 response governance. These flexibilites required a total of seven or eight assessments to be undertaken across the year, including the external examination. The school assessment component included:

* *five* or *six* skills and applications tasks
* *one mathematical investigation* task (with a maximum page limit of 15 A4 pages).

Teachers should refer to the subject outline for specifications on content and learning and assessment requirements, and to the subject operational information for operational matters and key dates.

School Assessment

Assessment Type 1: Skills and Application Tasks

Students complete six skills and application tasks under the direct supervision of the teacher. The equivalent of one skills and applications task must be undertaken without the use of either a calculator or notes.

The more successful responses commonly:

* included tasks that provided a mix of routine and more complex problems that effectively differentiated student mathematical knowledge and understanding of concepts and relationships across the grade bands
* provided evidence of the development and testing of a conjecture
* included tasks that provided the opportunity for interpretation of the mathematical results in the context of the problem
* included tasks that provided opportunity for the student to discuss logical conclusions from the mathematical results found
* used correct notation and terminology
* included tasks that allowed for multiple entry levels into extended questions by breaking it down into parts and including ‘show that‘ type questions.

The less successful responses commonly:

* were based on tasks that did not provide opportunity for students to solve more complex problems
* did not demonstrate an appropriate balance of understanding the key questions and concepts across the topics in the subject outline
* did not provided enough attention to detail in presenting their solutions
* lacked an understanding of how technology could be used effectively
* did not often attempt questions that asked for interpretation or logical conclusions of results
* were based on tasks that used questions directly from a textbook or from past exams
* were based on tasks that provided questions that were repetitive which either disadvantaged students with poor knowledge and understanding or advantaged students with highly effective knowledge and understanding.

Assessment Type 2: Investigation

Students complete one mathematical investigation with minimal teacher direction. The task should be written in a format that allows the student to conduct their own open-ended investigation. It must be completed in report format (if written) and must be no longer than 15 single sided A4 pages. Appendices should be used for repetitive calculations only.

The more successful responses commonly:

* included detailed development and application of a mathematical model
* demonstrated a comprehensive understanding of the reasonableness and limitations of the model developed, which can more easily be done in the context of a situation
* effectively communicated mathematical reasoning through the use of appropriate and well-labelled graphs and the inclusion of relevant calculations and notation
* made appropriate use of the appendices to avoid repetition in the report
* were based on tasks that included an element of student directed learning that differentiated student understanding of the model or problem being investigated.

The less successful responses commonly:

* included excessive routine and often repetitive calculations that were prescribed in the task design
* did not effectively select and apply appropriate mathematical techniques or use appropriate electronic technology when solving problems
* lacked depth in the interpretation of the mathematical results, often summarising what was calculated rather than analysing the results in the context of the problem
* were based on tasks that had a small open-ended component that students found difficult to further develop a mathematical model beyond fitting a curve.

External Assessment

Assessment Type 3: Examination

Examination markers aim to award marks for evidence of student understanding in responding to examination questions wherever possible, however, students should be advised not to cross out their responses or attempted responses to questions in the examination booklet, unless they are confident that no part of their response should be considered by the marker. Mathematical Methods external examinations will not award any marks to crossed-out work, so care should be taken by students when a decision is taken that work should not be considered. Overall, students appeared to find the examination to be approachable and they had success in accessing each question to some degree. However, with the reduction in time to two hours, evidence suggests that some students found the paper challenging to complete in the allocated time.

Question 1

This question provided an opportunity for students to demonstrate their routine calculus skills with 74% of students receiving eight or nine marks out of nine.

The more successful responses commonly:

* correctly implemented the chain rule, product rule and quotient rule when required
* remembered the constant of integration
* used the quotient rule in part (a) (iii) instead of rearranging the equation to become a product rule.

The less successful responses commonly:

* did not include appropriate brackets in their answers
* left out the in the numerator of the derivative of in part (a) (ii)
* used log laws in part (a) (ii) to place the in the exponent, creating a more complex derivative
* derived the expression instead of integrating it in part (b)
* did not include the negative coefficient arising from integrating the power of the exponential term in part (b).

Question 2

This question assessed the more routine aspects of probability density functions in conjunction with the application of differential calculus to find the maximum of a function. The question was not linked to any model or context in an attempt to allow more students to access the mathematics in the question; however, student’s success in this question varied greatly, with 37% gaining full marks, and 27% gaining three or fewer marks.

The more successful responses commonly:

* were selective in when and when not to use their graphics calculator throughout the question
* used factorisation to help solve when in part (d)
* considered the domain stated at the beginning of the question when finding the mode of the probability density function, leading to the removal of the negative solution to in part (d).

The less successful responses commonly:

* restated given information in part (a) (i.e. that the area under the curve was equal to one) or incorrectly stated that , confusing the probability density function with a probability of an event occurring
* did not successfully apply the product rule in part (c), leading to a simplification of part (c) and (d)
* did not take note of the specific wording used in part (d) (i.e. ‘exact’ and ‘algebraic approach’) when finding their answers.

Question 3

Students did well interpreting the context of this question with the majority of students achieving close to full marks. However, many students found it difficult to gain the final mark of the question as a result of not being concise enough in their reasoning.

The more successful responses commonly:

* correctly applied the relevant formula for the expected value of both a random variable modelled by a binomial distribution and a discrete random variable from the given table
* clearly and concisely linked the value of in part (b) (i) to the time taken for a robot *followed by* a human to attempt to retrieve the item.

The less successful responses commonly:

* stated properties of probability distributions that were too general in part (a) and not specific enough to a random variable that can be modelled by a Binomial Distribution. For example, ‘the probability of all events sum to 1’
* did not show logical steps of working when calculating probabilities, often resulting in the loss of multiple marks throughout the question
* incorrectly found the rather than in part (a) (iii), either resulting from students not reading the question carefully enough or treating as a continuous random variable
* did not supply sufficient evidence in their answer to part (c) that clearly showed that was a continuous random variable.

Question 4

This question contained a slightly more complex first principles question than often examined. Despite students generally achieving success in this question, there was a large variation in marks achieved across the cohort. Much of this variation was a result of students’ incorrect algebra or notation within the ‘first-principles’ part of the question. Additionally, students struggled to articulate their reasoning when assessing the ‘student statement’ in the final part of the question.

The more successful responses commonly:

* used correct limit notation in part (b)
* showed clear steps of logic in part (b). For example, students who showed:
  + clear substitution into the first principles formula,
  + clear cancellation of the before applying the limit,
  + clear evidence of ,
  + clear substitution of into ,

were able to demonstrate to the marker that was not found using direct differentiation or through their calculator.

The less successful responses commonly:

* unsuccessfully simplified the resulting fraction in part (a)
* tried to multiply by the conjugate when simplifying the fraction resulting from the first principles formula in part (b)
* did not sub in to find the derivative at the requested point in part (b)
* did not link the values they had calculated in parts (a) and (b) to their reasoning stated in their answer to part (c) (i.e. they didn’t link slopes of and , with the position of and in relation to ).

Question 5

This question explored the application of rectangles to approximate areas under a curved function in connection to how these estimations are linked to the true area underneath this specific function. Students coped well with this familiar question style with almost half of all students achieving 7 or 8 marks out of 8.

The more successful responses commonly:

* correctly identified both the width and height of the upper estimate in part (a)
* correctly identified the limits of the integral in part (c).

The less successful responses commonly:

* did not consider the word ‘exact’ when finding the upper estimate in part (a) and presented their answers and working in decimal form
* misinterpreted the use of the word ‘exact’ when finding the upper estimate in part (a) leading to students incorrectly using integrals
* incorrectly used and for the heights of their rectangles (i.e. they calculated the upper estimate) in part (a)
* did not supply enough evidence in part (b) that demonstrated that they could have independently found the average to be 24 (if it were not stated in the question)
* did not create a statement in part (d) which linked all four differences together (i.e. and only groups the differences in pairs and not all four together).

Question 6

This question contained mainly familiar statistical computations utilising graphics technology. Students generally demonstrated good skills in applying their knowledge of statistical concepts within the context given and were very successful in interpreting their calculated confidence interval. Approximately 3 quarters of students achieved half of the total marks or more.

The more successful responses commonly:

* clearly stated that the new confidence interval calculated in part (d) (ii) was *entirely above* the old mean value of
* used clear mathematical notation to communicate their methods, including identifying the various distributions being used throughout the question
* used technology to establish their confidence intervals.

The less successful responses commonly:

* incorrectly interpreted the probability that an individual oyster is between and as , showing a lack of understanding of the continuous nature of the weight of oysters in part (a)
* misinterpreted part (b) (ii) and found
* found the mean and standard deviation for in part (c) (ii) rather than
* did not use appropriate notation when stating their confidence intervals. For example, many students stated a confidence interval for , , or
* were not concise in discussing how their confidence calculated in part (d) (iii) was positioned in relation to the value of . For example, by only stating that was outside the confidence interval, this does not imply that the mean weight of oysters has increased (i.e. it could have decreased if was above the confidence interval)
* used the incorrect limit of their confidence interval to calculate the requested probability in part (d) (iii). Many students only calculated using the new sample mean (i.e. ) or the upper limit of their confidence interval calculated above in part (d) (i).

Question 7

This question focused on curve properties, drawing comparisons between the slopes and shapes of two functions. Students needed to be particularly careful with the signs and magnitudes of the first and second derivatives in order to obtain correct responses. Additionally, students needed to carefully link properties of various graphs to the first and second derivatives of functions. Generally speaking, students found this question challenging, despite its reasonably familiar structure, with only 5% of students achieving full marks.

The more successful responses commonly:

* identified the difference in properties of the first and second derivatives
* used all information given in the questions to successfully draw the graph of in part (e).

The less successful responses commonly:

* were not concise enough in the language used around stationary points and inflection points. Students were required to state that it was a local *minimum* and *stationary* point of inflection in parts (a) and (b) respectively
* misinterpreted/misread part (c) and stated where is increasing instead of
* did not consider that the inflection point corresponded to a maximum on their drawn graph of in part (e).

Question 8

This question contained a less-routine function with a conjecture-style question similar to many questions that have appeared in past exams. Students handled the concepts contained in this question well, resulting in a similar mean percentage correct to question 3 from booklet 1. Approximately one-quarter of students achieved full marks in this question, however, this question had the largest variation in marks of the paper.

The more successful responses commonly:

* clearly demonstrated the correct use of the product rule in part (a)
* were able to successfully differentiate the function containing and simplify to in part (c).

The less successful responses commonly:

* used decimal approximations in their answers to part (a) (ii)
* did not clearly show their algebraic steps in finding the equation of the tangent in both part (a) (ii) and part (c)
* showed that a new single individual integer case satisfied the conjecture in part (c).

Question 9

This question contained a complex context combining knowledge from Integral Calculus, and Samples and Confidence Intervals. Despite this question having multiple entry points, the data suggested that students found it difficult to apply their knowledge in this question with an overall mean of approximately 8 out of 16 marks.

The more successful responses commonly:

* correctly commented that the sample size was too small for a confidence interval to be established (due to the Central Limit Theorem) without relying on ‘rules of thumbs’ such as in part (a) (iii)
* identified that the confidence interval calculated in part (b) (ii) was for the proportion shaded, and not the area, hence requiring a conversion before comparisons could be made in part (b) (iii)
* used a ‘trial and error’ approach in part (b) (iv) showing the smallest value of (where is a positive integer) that created a confidence interval that supported the ‘students claim’, in conjunction with evidence that the confidence interval for did not support the claim.

The less successful responses commonly:

* constructed a confidence interval instead of a confidence interval throughout part (b)
* did not divide by the coefficient of (i.e. ) when integrating their answer to part (c) (i)
* finished with an answer larger than the area of the square (i.e. ) in part (c) (ii), however, did not comment on the inconsistent nature of this result.

Question 10

This question assessed concepts from Integral Calculus and Trigonometry Calculus. Students had to correctly interpret graphs and negotiate a difficult algebraic problem with unfamiliar trigonometric substitutions. As expected, this question had the lowest average mark in the paper with only 4% of students achieving full marks. Although the lower average result can be partly explained by students not managing their time successfully, it was also due to students finding it difficult to approach the complex processes in the final two parts of the question. These questions required careful algebraic working and in-depth understanding of optimising functions.

The more successful responses commonly:

* correctly stated the corresponding values for both maximums in part (a) (ii)
* carefully considered rounding throughout the question
* appreciated that, when working with trigonometric functions, there is often more than one solution and, in this case, the ‘second solution’ was required in part (c) (ii)
* used a sign diagram to demonstrate that their value of in part (c) (ii) was a minimum.

The less successful responses commonly:

* did not shade in the area in part (b), despite drawing in the two vertical lines at and
* did not show clear and logical steps in finding the given function in part (c) (i)
* incorrectly let or in part (c) (ii).