**Stage 2 Mathematical Methods**

**Program 2**

This example program covers the following topics based on a four-lesson week format (two single lessons and one double lesson). There is additional time at the end of the program to allow for interruptions to occur or extra time spent on topics if required.

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| **Term 1** | **Term 2** | **Term 3** | **Term 4** |
| Further Differentiation and Applications  Logarithmic Functions | Continuous Random Variables and the Normal Distribution  Sampling and Confidence Intervals  Discrete Random Variables | Integral Calculus | Exam Revision |

Many topics and subtopics specified in the program may be revisited or covered several times through the program. The topics and subtopics are not exclusive to those lessons or weeks as the placement of content and assessment tasks within this program is a guide only.

Students demonstrate their learning through six Skills and Applications Tasks (50%) and one Mathematical Investigation (20%) for the school assessment, as specified in the Mathematical Methods subject outline for teaching in 2017.

| **Week** | **Single Lesson** | **Double Lesson** | **Single Lesson** |
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| Term 1  Week 1 | ***TOPIC 1: Further Differentiation and Applications***  *Subtopic 1.1: Introductory Differential Calculus*  **Rates of change**  Instantaneous rate of change  Increasing and decreasing rates of change | **Limits and differentiation from first principles**  Slope of a tangent  First principles  Derivative of polynomial functions and power functions  Displacement and velocity  Applications to modelling  Local maxima and minima | *Subtopic 1.2: Differentiation Rules*  **Derivative of**  Simple rules of differentiation  If  Then, |
| 2 | **Derivatives of functions with negative and fractional indices**  Derivative of  Review indices laws including | **Composite Functions and the Chain Rule**  Chain rule  Problems involving | **The Product and Quotient Rule**  Product rule  Quotient rule |
| 3 | **The Product and Quotient Rule**  Continuation from previous lesson | *Subtopic 1.5: The Second Derivative*  **The Second Derivative**  The relationship between the function, its derivative, and the second derivative from graphical examples | **Curve Properties**  Increasing and decreasing functions  Revision of intervals  Increasing and decreasing intervals |
| 4 | **Curve Properties**  Stationary points  Maxima/Minima  Inflection points  Curve sketching including concavity and points of inflection | **Review** | **SAT 1**  Differential Calculus Test  (No calculators or notes) |
| 5 | **Motion in a Straight Line**  Displacement, acceleration, and velocity | **Motion in a Straight Line**  Average acceleration  Instantaneous acceleration  Speed (increasing / decreasing)  Sign diagrams  Motion diagrams | **Optimisation**  Examples of optimisation |
| 6 | *Subtopic 1.3: Exponential Functions*  **The derivative of and y** | **Exponential, Surge and Logistic Modelling**  Growth and decay functions  Surge functions  Logistic functions | **Using exponential functions**  Slope of tangents to graphs of functions  Local maxima and minima  Increasing and decreasing functions  Displacement and velocity  Applications to model actual scenarios using exponential functions |
| 7 | ***TOPIC 4: Logarithmic Functions***  *Subtopic 4.1: Using Logarithms for Solving Exponential Equations*  **Solving Exponential Equations**  Given then leading to  when then  Solving exponential equations and the revision of the log laws | *Subtopic 4.2: Logarithmic Functions and their Graphs*  **Graphs**  Logarithmic scale  The graph of and its properties  The graph of functions in the form  The relationship between the graphs of and | *Subtopic 4.3: Calculus of Logarithmic Functions*  **The derivatives of and**  provided is positive  Problem solving using the derivatives of logarithmic functions  Applications of logarithmic functions. |
| 8 | **Applications of Exponential and Logarithmic Functions**  Continuation of previous lessons | **Mathematical Investigation**  Surge and Logistic Models | **Mathematical Investigation**  Surge and Logistic Models |
| 9 | **SAT 2**  Applications of Differential Calculus Test | **Mathematical Investigation**  Surge and Logistic Models | **Mathematical Investigation**  Surge and Logistic Models |
| 10 | *Subtopic 1.4: Trigonometric Functions*  **Review of Radians and the Unit Circle**  Symmetry and exact values  e.g.  Unit circle quadrants (ASTC)  Negative angles  Complementary angles  Special cases | **Trigonometric equations and graphs**  General solutions of trigonometric equations  Graphs of sine and cosine functions  Dilation  Reflection  Translation  Period  Amplitude | **Trigonometric Modelling**  Using functions like  to model periodic behaviour  **Mathematical Investigation**  Submission |
| 11 | **Derivatives of**  Use the product rule to find derivative of | **Derivatives of**  Applying the chain rule to trigonometric functions  and | **Using the product and quotient rules with trigonometric functions**  Derivatives of functions such as: |
| Term 2  Week 1 | **Using derivatives of trigonometric functions**  Review  Using trigonometric functions  Slope of tangents to graphs of functions  Local maxima and minima  Increasing and decreasing functions  Displacement and velocity | **Review**  Modelling periodic scenarios such as tidal heights, temperature changes and AC voltages | **SAT 3**  Further Differentiation Test |
| 2 | ***TOPIC 5: Continuous Random Variables and the Normal Distribution***  *Subtopic 5.1: Continuous Random Variables*  **Continuous Random Variables**  Comparing discrete and continuous random variables  The probability of a specific range of values | **Probability density functions**  Probability density functions and their graphs  The mean from and the  standard deviation | *Subtopic 5.2: Normal Distributions*  **The Normal Distribution**  Standard deviation  Conditions for a normal random variable  The key properties of normal distributions  The probability density function  Using electronic technology to calculate proportions, probabilities, and the upper or lower limit of a certain proportion |
| 3 | The standard normal distribution with and  Standardising a normal distribution using | *Subtopic 5.3: Sampling*  **Distribution of Sample Means**  For a sample of a random variable  **Sampling Distributions**  the outcome of adding n independent observations of *X*  the outcome of averaging *n* independent observations of *X*  If then and  provided *n* is sufficiently large.\* | Simple random sample  If then for a sample size  Central limit theorem  \* using the notation for a normal distribution with mean and standard deviation |
| 4 | **Sampling**  Continuation from previous lesson | ***TOPIC 6: Sampling and Confidence Intervals***  *Topic 6.1: Confidence Intervals for Population Mean*  **Confidence Intervals for Means**  Sample means are continuous random variables  Distribution of sample means will be approximately normal for a sufficiently large sample.  A confidence interval can be created around the sample mean that may contain the population mean. If is the sample mean then the CI is , where is determined by the confidence level that the interval will contain the population mean. | **Confidence Intervals for Means**  e.g. 95% confidence intervals  Increasing sample size decreases interval width  Finding the sample mean and standard deviation based on a confidence interval |
| 5 | **Confidence Intervals for Means**  Using a confidence interval for a claim  Assessing a claim about a population mean  Other applications of confidence intervals  Confidence intervals other than 95% | **Confidence Intervals for Means**  Continuation from previous lesson | **Review** |
| 6 | **Review** | **SAT 4**  Normal Distribution Test | ***TOPIC 2: Discrete Random Variables***  *Subtopic 2.1: Discrete Random Variables*  **Discrete Random Variables**  Discrete vs. continuous  Probability distributions  Uniform/non-uniform discrete random variables  Expected value  Standard deviation of a discrete random variable |
| 7 | **Discrete Random Variables**  Continuation from previous lesson | *Subtopic 2.2: The Bernoulli Distribution*  **The Bernoulli Distribution**  Bernoulli variables are discrete random variables with only two outcomes; success or failure  The Bernoulli distribution  The mean and standard deviation.  Assigning Probabilities  i.e. correctly in probability functions | *Subtopic 2.3: Repeated Bernoulli Trials and the Binomial Distribution*  The binomial random variable and the binomial distribution  The mean and the standard deviation  Modelling scenarios using the binomial distribution.  Finding binomial probabilities using  and electronic technology  The shape of the binomial distribution for large values of  **The Binomial Probability Density Function and the Binomial Cumulative Density Function**  and  Finding probabilities greater than using the binomcdf function |
| 8 | *Subtopic 6.2: Population Proportions*  **Population Proportions**  Concept of a population proportion .  Sample proportion  mean and standard deviation .  As the sample size increases the distribution of becomes more like a normal distribution | **Population Proportions**  Continuation from previous lesson | *Subtopic 6.3: Confidence Intervals for Population Proportions*  **Confidence Intervals for Proportions**  is the sample mean; the confidence interval is  , where is determined by the confidence that the interval will contain the population mean.  e.g. 95% confidence intervals for proportions |
| 9 | **Confidence Intervals for Proportions**  Continuation from previous lesson | **Assessing Claims with Confidence Intervals**  What does it mean if *p* falls within/outside the confidence interval?  How does the size of the sample affect the confidence interval width? | **Assessing Claims with Confidence Intervals**  Continuation from previous lesson |
| 10 | **Review** | **SAT 5**  Discrete Random Variables and Proportions Test | ***TOPIC 3: Integral Calculus***  *Subtopic 3.1: Anti-differentiation*  **Anti-Differentiation**  Changing a derivative to the original function  The antiderivative of  *The indefinite integral ; and*  *Integrals of including consideration of functions of the form )*  *These points are covered in more detail in the program in the weeks below.* |
| Term 3  Week 1 | **Anti-Differentiation**  Revision and continuation from previous lesson  Using  Determining the specific constant of integration. | *Subtopic 3.2: The Area under Curves*  **Definite Integrals**  Estimating with lower and upper sum  Strategies to improve the estimate.  Use of electronic technology to find the area.  Properties of definite integrals | **Definite Integrals**  Using the terminology for the exact area for a positive continuous function, *−* for a negative continuous function, and for the area between two curves where is above  The observations  *=* 0 and |
| 2 | *Subtopic 3.3: Fundamental Theorem of Calculus*  **The Fundamental Theorem of Calculus**  Know that  and, | **The Fundamental Theorem of Calculus**  Continuation from previous lesson  Evaluating the exact area under a curve and the area between two curves. | **The Indefinite Integral**  Discovering integrals by differentiation |
| 3 | **Integration**  Simple Integrals | **Integration**  Continuation from previous lesson | **Integration**  Integrating |
| 4 | **Integration**  Continuation of previous lesson | **Integration of**  Mixed problems with | **Integration of** |
| 5 | **Linear Motion (Integration)**  Total distance travelled vs. Displacement  The Absolute Value | **Linear Motion (Integration)**  Continuation from the previous lesson | **Definite Integrals** |
| 6 | **Definite Integrals**  Continuation of previous lesson | *Subtopic 3.4: Applications of Integration*  **Finding Areas**  Area of region bounded by  Area between two functions  The area of cross-sections | **Applications of Definite Integration**  The total change in a quantity given the rate of change over a time period. |
| 7 | **Applications of Definite Integration**  Distance from velocity-time | **Further Applications**  For example:   * the rate at which people enter a sports venue * water flow during a storm * traffic flow * electricity consumption | **Review** |
| 8 | **Review** | **Review** | **SAT 6**  Integral Calculus Test |
| 9/10 | **Exam Revision / Time for adjustment to program due to interruptions, excursions, etc.** | | |
| Term 4 | **Revision / SWOT VAC / Examination** | | |