Stage 2 Specialist Mathematics

Topic 1: Mathematical Induction, Topic 2: Complex Numbers, Topic 3: Functions and Sketching Graphs, Topic 4: Vectors in 3D, Topic 5: Integration Techniques and Applications, and Topic 6: Rates of Change and Differential Equations.

**Important Notes**

* **The placement of content and assessment tasks within this program is a guide only.**
* **Excursions, sports days and other co- and extra-curricular activities is a consideration that is not part of this program.**
* **School Assessment: six Skills and Applications Tasks (50%) and one Mathematical Investigation (20%).**

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| **Lesson 1 – Single Lesson** | **Lesson 2, 3 – Single Lessons** | **Lesson 4 – Double Lesson** |
| **Course Overview and Expectations****TOPIC THREE: FUNCTIONS AND SKETCHING GRAPHS** **Sub-topic 3.1 Composition of Functions** Consider composite functions and the requirements on domain and range relationships. | Finding compositions:* Check appropriate domains.
 | Application of concepts to further problems. |
| **Sub-topic 3.2 One-to-One Functions** Determine if a function is one-to-one:* only when
* Horizontal line test.
 | Inverse  of a one-to-one function:* Unique value of domain corresponding to each element of range.

Determine the inverse of a one-to-one function.Relationship between a function and the graph of its inverse. | Investigate symmetry about :* Software or graphics calculator approach
* Note relationship between exponential and log functions (cf. Mathematical Methods).
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| **Sub-topic 3.3 Sketching Graphs** Absolute value function notation and properties.Composite functions with absolute value: * and
 | Reciprocal functions  where is linear, quadratic or trigonometric. | Graphs of rational functions:* Numerator and denominator both up to degree 2 with real zeros
* Asymptotic behavior via graphics calculator or other technology.
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| Revision – Functions and Sketching Graphs. | **TOPIC ONE: MATHEMATICAL INDUCTION****Sub-topic 1.1 Proof by Mathematical Induction**Understanding inductive proof with the initial statement and inductive step (revision of concept from Stage 1 Mathematics). | **SAT 1 – Functions and Sketching Graphs**  |
| Mathematical Induction continued:* Formal proofs using mathematical induction (including initial statement and inductive step).
 | Mathematical Induction continued. | **TOPIC TWO: COMPLEX NUMBERS** **Sub-topic 2.1 Cartesian and Polar Form** Review of complex numbers:* Cartesian form
* Real and imaginary parts.

Arithmetic of complex numbers. |
| Describe sets of points in the complex plane:* Circular regions
* Rays from the origin.
 | **SAT 2 – Mathematical Induction (no ET)** Convert from Cartesian form to polar form* Algebraic approach
* Calculator approach
 | Properties of complex numbers:*

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Multiplication by  * Dilation by r
* Rotation by θ
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| Multiplication by  continued:Examples involving Stage 1 Geometry: e.g. look at rhombus properties to find  from . | Use the Principle of Mathematical Induction to prove:*
* .
 | Prove and use de Moivre’s Theorem.Consider previous proofs where all θ are equal and special case of  all equal. |
| de Moivre’s theorem problems:* Further examples of using the theorem.
 | Negative powers and fractional powers. | Continue with problem solving using polar form and de Moivre’s theorem. |

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| **Sub-topic 2.2 The Complex (Argand) Plane** Addition of complex numbers – vector addition on complex plane.Multiplication of complex numbers (polar form):* Multiplying w by
* Multiplying by  and noting the rotation.
 | Multiplication continued.Distance between points in the complex plane:* Geometrically
* Triangle Inequality for the sum of the lengths of complex numbers

 e.g. * Consider also the situation of more than three sides. The possibility for another PMI proof.
 | Continuation of previous lessons.Geometrical interpretation of equations and inequalities:* Circles, lines, rays, regions

Continuation of geometric interpretation problems. |
| Cartesian equations formed for some cases.Polar graphs. | Continuation of previous lessons.* Geometry software
* Graphics calculators
 | **Sub-topic 2.3 Roots of Complex Numbers** Solving  with c complex. |
| Consider  * nth roots of unity on the Argand Plane

Sum of roots by vectors – construction of n-gon. | Application of concepts to further problems. | **Sub-topic 2.4 Factorisation of Polynomials** Review of multiplying polynomials.Long division or synthetic division. |
| Equating coefficients when one factor given to lead to factorisation of polynomial.Roots, zeros, factors. | Prove and apply Factor and Remainder Theorems.Verifying zeros. | Factorising real cubics and quartics using complex roots and their conjugates. |
| Zeros and shape of curves.Special examples using de Moivre’s Theorem.Factorisation of  | Application of concepts to further problems.Revision. | **SAT 3 – Complex Numbers** |
| **TOPIC FOUR: VECTORS IN 3D** **Sub-topic 4.1 The Algebra of Vectors in 3D** Review of vectors from Stage 1 Topic 3.Developing to 3D. | Unit vectors:*
 | Continue algebra of vectors in 3D. |

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| **Sub-topic 4.2 Vector and Cartesian Equations** Cartesian coordinates:* plotting points
* equations of spheres

Equation of a line in 2D and 3D:* vector equation and parametric form.
 | * Cartesian form
* Parallel, perpendicular and skew lines
 | * Closest point on line to another point
* Distance between skew lines
* Angle between two lines.
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| Path of two particles:* Using vectors as functions of time, do particles paths cross or meet?
 | **Investigation – The Problem with Mosquitoes** | **Investigation – The Problem with Mosquitoes** |
| Scalar (dot) product and vector (cross) product:* Coordinates, length and angle

Context: Perpendicular and parallel vectors | Vector (cross) product:* calculation using the determinant (2x2 and 3x3).
* geometric relevance:  is the area of a parallelogram, sides **a** and **b**.
 | Equation of a plane:* Develop using vector equations
* Intersection of a line and a plane.

Lines parallel to or coincident with planes. |
| Find the point on a given plane closest to a point in space.Equality of vectors:* Seen using opposite sides of parallelogram

Coordinate systems and position vectorsTriangle Inequality:* Connection from Sub-topic 2.2
 | Vector Proof:* Establishing parallelism, perpendicularity, properties of intersections

If where  are not parallel, then  | **Sub-topic 4.3 Systems of Linear Equations** General form of system of equations:* Elementary techniques of elimination to solve up to 3x3 system.
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| Application of concepts to further problems. | Possible solutions and geometric interpretation (continued).* Algebraic and geometric descriptions of:
* unique solution
* no solution
* infinitely many solutions
 | Finding intersection of two or more planes. |

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| **TOPIC FIVE: INTEGRATION TECHNIQUES AND APPLICATIONS** **Sub-topic 5.1 Integration Techniques** Integration of trigonometric and composite functions* Use of trigonometric identities
* Substitution method
 | Integration of trigonometric and composite functions (continued).* Establish and use , for
 | **SAT 4 – Vectors in 3D**Application of concepts to further problems. |
| Application of concepts to further problems. | Find and use the inverse trigonometric functions:* Restricted domain to obtain one-to-one functions.
 | Find and use the derivatives of inverse trigonometric functions:* Integrate expressions of the form
 |
| Previous lesson continued. | Use partial fractions for integrating simple rational functions. | Use partial fractions for integrating simple rational functions continued.Use integration by parts:. |
| Previous lesson continued. | **Sub-topic 5.2 Applications of Integral Calculus** Areas between curves. | Volumes of revolution* About the x axis
* About the y axis
 |
| Volumes of revolution (continued)* Graphical approach for derivation of formulae
 | Revision. | **SAT 5 – Integration Techniques and Applications** |
| **TOPIC SIX: RATES OF CHANGE AND DIFFERENTIAL EQUATIONS** **Sub-topic 6.1 Implicit Differentiation**Implicit differentiation (following on from Mathematical Methods) | Finding gradients of curves in implicit form.Derivation of the derivative of the natural log function. | **Sub-topic 6.2 Differential Equations** Related Rates.* Examples of calculating from
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| Related Rates (continued):Differential Equations:* Solving
 | Differential Equations:* Solving
* Solving
 | Application of concepts to further problems. |
| Slope fields:* For first-order differential equations
* Graph from slope field manually and using software or calculators.
 | Previous lesson continued. | Modelling with differential equations:* Separable differential equation examples
* Logistic
 |
| Application of concepts to further problems. | **Sub-topic 6.3 Pairs of Varying Quantities –Polynomials of Degree 1 to 3**Curves produced by moving point Coordinate representation (parametric):* Quantities of the form
 | Vector representation with *t* as time:*

Examples:* Objects in free flight.
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| Application – Bézier curves | Application – Bézier curves | **Sub-topic 6.4 Related Rates, Velocity and Tangents** For a moving point :* **V** =is the instantaneous velocity vector.

Cartesian equation from parametric. |
| Velocity vector is tangent to the curve.Parametric equations of tangents to parametric curves. | * Speed of moving point

 * Arc length of path traced out.
 | **Sub-topic 6.5 Trigonometric Parameterisations** * Point moving with unit speed around the unit circle has position
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| Consider :* Use Arc length formula to establish circumference of a circle.
 | Revision. | **SAT 6: Rates of Change and Differential Calculations** |