Stage 2 Specialist Mathematics

Topic 1: Mathematical Induction, Topic 2: Complex Numbers, Topic 3: Functions and Sketching Graphs, Topic 4: Vectors in 3D, Topic 5: Integration Techniques and Applications, and Topic 6: Rates of Change and Differential Equations.

**Important Notes**

* **The placement of content and assessment tasks within this program is a guide only.**
* **Excursions, sports days and other co- and extra-curricular activities is a consideration that is not part of this program.**
* **School Assessment: six Skills and Applications Tasks (50%) and one Mathematical Investigation (20%).**

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| **Lesson 1 – Single Lesson** | **Lesson 2, 3 – Single Lessons** | **Lesson 4 – Double Lesson** |
| **Course Overview and Expectations**  **TOPIC THREE: FUNCTIONS AND SKETCHING GRAPHS**  **Sub-topic 3.1 Composition of Functions**  Consider composite functions and the requirements on domain and range relationships. | Finding compositions:   * Check appropriate domains. | Application of concepts to further problems. |
| **Sub-topic 3.2 One-to-One Functions**  Determine if a function is one-to-one:   * only when * Horizontal line test. | Inverse  of a one-to-one function:   * Unique value of domain corresponding to each element of range.   Determine the inverse of a one-to-one function.  Relationship between a function and the graph of its inverse. | Investigate symmetry about :   * Software or graphics calculator approach * Note relationship between exponential and log functions (cf. Mathematical Methods). |
| **Sub-topic 3.3 Sketching Graphs**  Absolute value function notation and properties.  Composite functions with absolute value:   * and | Reciprocal functions    where is linear, quadratic or trigonometric. | Graphs of rational functions:   * Numerator and denominator both up to degree 2 with real zeros * Asymptotic behavior via graphics calculator or other technology. |

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| Revision – Functions and Sketching Graphs. | **TOPIC ONE: MATHEMATICAL INDUCTION**  **Sub-topic 1.1 Proof by Mathematical Induction**  Understanding inductive proof with the initial statement and inductive step (revision of concept from Stage 1 Mathematics). | **SAT 1 – Functions and Sketching Graphs** |
| Mathematical Induction continued:   * Formal proofs using mathematical induction (including initial statement and inductive step). | Mathematical Induction continued. | **TOPIC TWO: COMPLEX NUMBERS**  **Sub-topic 2.1 Cartesian and Polar Form**  Review of complex numbers:   * Cartesian form * Real and imaginary parts.   Arithmetic of complex numbers. |
| Describe sets of points in the complex plane:   * Circular regions * Rays from the origin. | **SAT 2 – Mathematical Induction (no ET)**  Convert from Cartesian form to polar form   * Algebraic approach * Calculator approach | Properties of complex numbers:          Multiplication by   * Dilation by r * Rotation by θ |
| Multiplication by  continued:  Examples involving Stage 1 Geometry: e.g. look at rhombus properties to find  from . | Use the Principle of Mathematical Induction to prove:    * . | Prove and use de Moivre’s Theorem.  Consider previous proofs where all θ are equal and special case of  all equal. |
| de Moivre’s theorem problems:   * Further examples of using the theorem. | Negative powers and fractional powers. | Continue with problem solving using polar form and de Moivre’s theorem. |

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| **Sub-topic 2.2 The Complex (Argand) Plane**  Addition of complex numbers – vector addition on complex plane.  Multiplication of complex numbers (polar form):   * Multiplying w by * Multiplying by  and noting the rotation. | Multiplication continued.  Distance between points in the complex plane:   * Geometrically * Triangle Inequality for the sum of the lengths of complex numbers   e.g.   * Consider also the situation of more than three sides. The possibility for another PMI proof. | Continuation of previous lessons.  Geometrical interpretation of equations and inequalities:   * Circles, lines, rays, regions   Continuation of geometric interpretation problems. |
| Cartesian equations formed for some cases.  Polar graphs. | Continuation of previous lessons.   * Geometry software * Graphics calculators | **Sub-topic 2.3 Roots of Complex Numbers**  Solving  with c complex. |
| Consider   * nth roots of unity on the Argand Plane   Sum of roots by vectors – construction of n-gon. | Application of concepts to further problems. | **Sub-topic 2.4 Factorisation of Polynomials**  Review of multiplying polynomials.  Long division or synthetic division. |
| Equating coefficients when one factor given to lead to factorisation of polynomial.  Roots, zeros, factors. | Prove and apply Factor and Remainder Theorems.  Verifying zeros. | Factorising real cubics and quartics using complex roots and their conjugates. |
| Zeros and shape of curves.  Special examples using de Moivre’s Theorem.  Factorisation of | Application of concepts to further problems.  Revision. | **SAT 3 – Complex Numbers** |
| **TOPIC FOUR: VECTORS IN 3D**  **Sub-topic 4.1 The Algebra of Vectors in 3D**  Review of vectors from Stage 1 Topic 3.  Developing to 3D. | Unit vectors: | Continue algebra of vectors in 3D. |

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| **Sub-topic 4.2 Vector and Cartesian Equations**  Cartesian coordinates:   * plotting points * equations of spheres   Equation of a line in 2D and 3D:   * vector equation and parametric form. | * Cartesian form * Parallel, perpendicular and skew lines | * Closest point on line to another point * Distance between skew lines * Angle between two lines. |
| Path of two particles:   * Using vectors as functions of time, do particles paths cross or meet? | **Investigation – The Problem with Mosquitoes** | **Investigation – The Problem with Mosquitoes** |
| Scalar (dot) product and vector (cross) product:   * Coordinates, length and angle   Context: Perpendicular and parallel vectors | Vector (cross) product:   * calculation using the determinant (2x2 and 3x3). * geometric relevance:  is the area of a parallelogram, sides **a** and **b**. | Equation of a plane:   * Develop using vector equations * Intersection of a line and a plane.   Lines parallel to or coincident with planes. |
| Find the point on a given plane closest to a point in space.  Equality of vectors:   * Seen using opposite sides of parallelogram   Coordinate systems and position vectors  Triangle Inequality:   * Connection from Sub-topic 2.2 | Vector Proof:   * Establishing parallelism, perpendicularity, properties of intersections   If where  are not parallel, then | **Sub-topic 4.3 Systems of Linear Equations**  General form of system of equations:   * Elementary techniques of elimination to solve up to 3x3 system. |
| Application of concepts to further problems. | Possible solutions and geometric interpretation (continued).   * Algebraic and geometric descriptions of: * unique solution * no solution * infinitely many solutions | Finding intersection of two or more planes. |

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| **TOPIC FIVE: INTEGRATION TECHNIQUES AND APPLICATIONS**  **Sub-topic 5.1 Integration Techniques**  Integration of trigonometric and composite functions   * Use of trigonometric identities * Substitution method | Integration of trigonometric and composite functions (continued).   * Establish and use , for | **SAT 4 – Vectors in 3D**  Application of concepts to further problems. |
| Application of concepts to further problems. | Find and use the inverse trigonometric functions:   * Restricted domain to obtain one-to-one functions. | Find and use the derivatives of inverse trigonometric functions:   * Integrate expressions of the form |
| Previous lesson continued. | Use partial fractions for integrating simple rational functions. | Use partial fractions for integrating simple rational functions continued.  Use integration by parts:. |
| Previous lesson continued. | **Sub-topic 5.2 Applications of Integral Calculus**  Areas between curves. | Volumes of revolution   * About the x axis * About the y axis |
| Volumes of revolution (continued)   * Graphical approach for derivation of formulae | Revision. | **SAT 5 – Integration Techniques and Applications** |
| **TOPIC SIX: RATES OF CHANGE AND DIFFERENTIAL EQUATIONS**  **Sub-topic 6.1 Implicit Differentiation**  Implicit differentiation (following on from Mathematical Methods) | Finding gradients of curves in implicit form.  Derivation of the derivative of the natural log function. | **Sub-topic 6.2 Differential Equations**  Related Rates.   * Examples of calculating from |
| Related Rates (continued):  Differential Equations:   * Solving | Differential Equations:   * Solving * Solving | Application of concepts to further problems. |
| Slope fields:   * For first-order differential equations * Graph from slope field manually and using software or calculators. | Previous lesson continued. | Modelling with differential equations:   * Separable differential equation examples * Logistic |
| Application of concepts to further problems. | **Sub-topic 6.3 Pairs of Varying Quantities –Polynomials of Degree 1 to 3**  Curves produced by moving point  Coordinate representation (parametric):   * Quantities of the form | Vector representation with *t* as time:      Examples:   * Objects in free flight. |
| Application – Bézier curves | Application – Bézier curves | **Sub-topic 6.4 Related Rates, Velocity and Tangents**  For a moving point :   * **V** =is the instantaneous velocity vector.   Cartesian equation from parametric. |
| Velocity vector is tangent to the curve.  Parametric equations of tangents to parametric curves. | * Speed of moving point      * Arc length of path traced out. | **Sub-topic 6.5 Trigonometric Parameterisations**   * Point moving with unit speed around the unit circle has position |
| Consider :   * Use Arc length formula to establish circumference of a circle. | Revision. | **SAT 6: Rates of Change and Differential Calculations** |