Performance Standards for Stage 1 General Mathematics

overall grade B

	Concepts and Techniques	Reasoning and Communication
A	Comprehensive knowledge and understanding of concepts and relationships. Highly effective selection and application of mathematical techniques and algorithms to find efficient and accurate solutions to routine and complex problems in a variety of contexts. Successful development and application of mathematical models to find concise and accurate solutions. Appropriate and effective use of electronic technology to find accurate solutions to routine and complex problems.	Comprehensive interpretation of mathematical results in the context of the problem. Drawing logical conclusions from mathematical results, with a comprehensive understanding of their reasonableness and limitations. Proficient and accurate use of appropriate mathematical notation, representations, and terminology. Highly effective communication of mathematical ideas and reasoning to develop logical and concise arguments. Formation and testing of appropriate predictions, using sound mathematical evidence.
B	Some depth of knowledge and understanding of concepts and relationships. Mostly effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine and some complex problems in a variety of contexts. Attempted development and successful application of mathematical models to find mostly accurate solutions. Mostly appropriate and effective use of electronic technology to find mostly accurate solutions to routine and some complex problems.	Mostly appropriate interpretation of mathematical results in the context of the problem. Drawing mostly logical conclusions from mathematical results, with some depth of understanding of their reasonableness and limitations. Mostly accurate use of appropriate mathematical notation, representations, and terminology. Mostly effective communication of mathematical ideas and reasoning to develop mostly logical arguments. Formation and testing of mostly appropriate predictions, using some mathematical evidence.
C	Generally competent knowledge and understanding of concepts and relationships. Generally effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine problems in different contexts. Application of mathematical models to find generally accurate solutions. Generally appropriate and effective use of electronic technology to find mostly accurate solutions to routine problems.	Generally appropriate interpretation of mathematical results in the context of the problem. Drawing some logical conclusions from mathematical results, with some understanding of their reasonableness and limitations. Generally appropriate use of mathematical notation, representations, and terminology, with reasonable accuracy. Generally effective communication of mathematical ideas and reasoning to develop some logical arguments. Formation of an appropriate prediction and some attempt to test it using mathematical evidence.
D	Basic knowledge and some understanding of concepts and relationships. Some selection and application of mathematical techniques and algorithms to find some accurate solutions to routine problems in context. Some application of mathematical models to find some accurate or partially accurate solutions. Some appropriate use of electronic technology to find some accurate solutions to routine problems.	Some interpretation of mathematical results. Drawing some conclusions from mathematical results, with some awareness of their reasonableness. Some appropriate use of mathematical notation, representations, and terminology, with some accuracy. Some communication of mathematical ideas, with attempted reasoning and/or arguments. Attempted formation of a prediction with limited attempt to test it using mathematical evidence.
E	Limited knowledge or understanding of concepts and relationships. Attempted selection and limited application of mathematical techniques or algorithms, with limited accuracy in solving routine problems. Attempted application of mathematical models, with limited accuracy. Attempted use of electronic technology, with limited accuracy in solving routine problems.	Limited interpretation of mathematical results. Limited understanding of the meaning of mathematical results, their reasonableness or limitations. Limited use of appropriate mathematical notation, representations, or terminology, with limited accuracy. Attempted communication of mathematical ideas, with limited reasoning. Limited attempt to form or test a prediction.

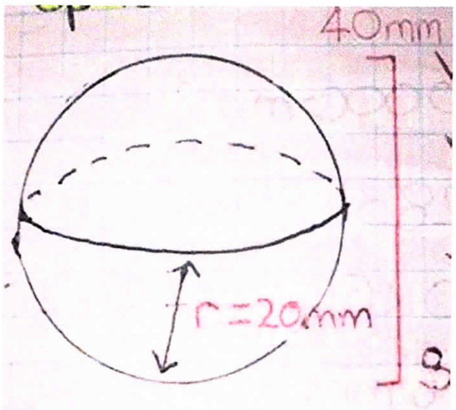
Ref: A576960 (created October 2016)

Stage One: Mathematical Investigation - Box of Chocolates

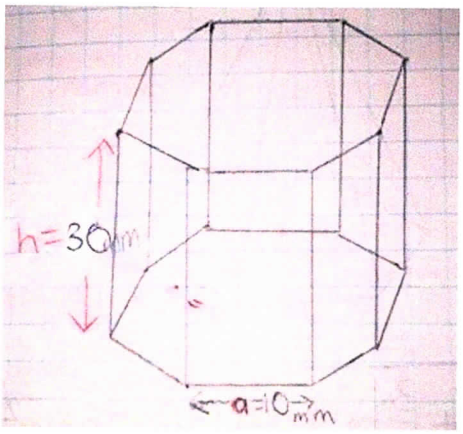
In this investigation, three separate pieces of chocolate and the packaging are designed, consisting of one simple standard shape and two complex solid shapes. The two complex designs require two types of shapes to be utilized, planar faces and curved surfaces. This investigation tests measurement skills learnt throughout the term. Word has been used to layout the 5 parts of the investigation. The five parts comprise of the design of the three unique shapes, the volume, surface area and costing of the tray and packaging box and a conclusion, all parts have been clearly depicted with all working out and sketches.

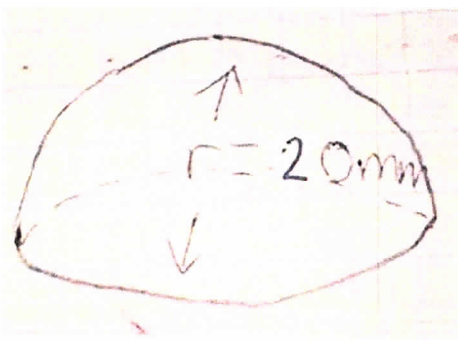
Part One – The Design

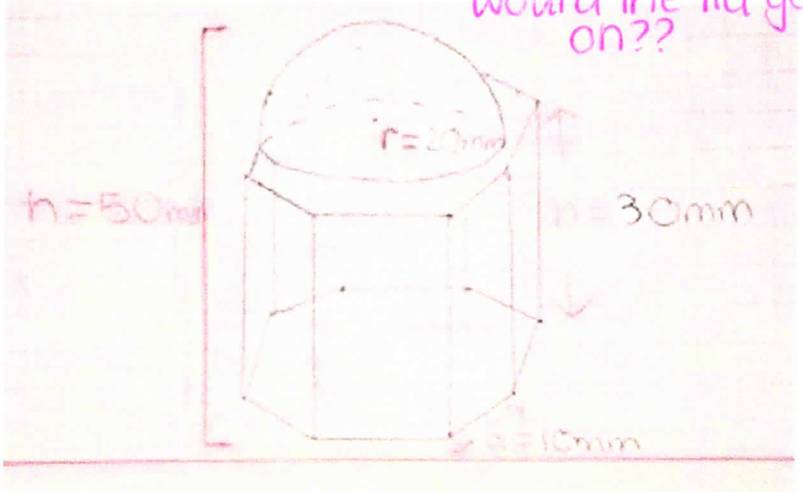
→ Simple Design – **Sphere**



→ Complex Design – **Octagonal Prism & Semi-sphere**

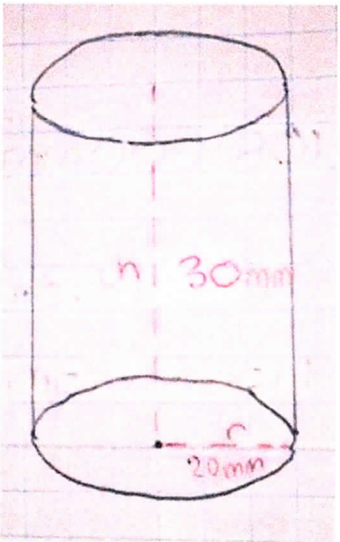


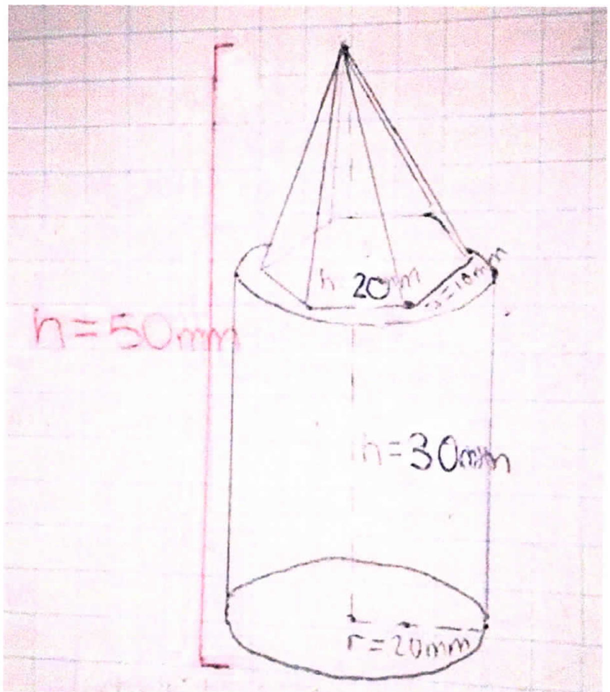


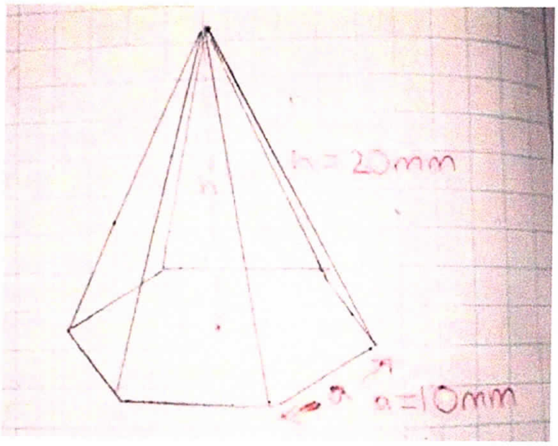


The height of your chocolate is 50mm and so is the box - is this really practical, would the lid go on??

→ Complex Design – **Cylinder & Right Hexagonal Pyramid**







↑ Once again chocolate height is the same as box height.

Part Two – Estimating and Calculating Volume

a) Estimating Volume

Estimations were made by manipulating other formulas for other shapes and replacing π with 3. Then true volume was calculated, followed by calculating absolute and percentage error.

→ **Sphere**

VA = 4/3 x 3 x 20 x 20 x 20 = 32000 mm^3 ✓

← Would have been clearer to show formula e.g $\frac{4}{3} \times \pi \times r^3$

→ **Octagonal Prism & Semi Sphere**

Octagonal Prism

VA = 2 x 2 x 10^2 x 30 = 12000 mm^3

Semi Sphere

VA = 4/3 x 3 x 20 x 20 x 20 = 32000 ÷ 2 = 16000 mm^3

Total = 12000 + 16000 = 28000mm^3 ✓

← For estimation you were meant to use a simple shape such as cylinders

(interpretation e.g the actual volume of shape 1 is 28000mm^3)

→ **Cylinder & Hexagonal Pyramid**

Cylinder

VA = 3 x 20^2 x 30 = 36000 mm^3

Hexagonal Pyramid

VA = ½ x 10^2 x 20 = 1000 mm^3

← A cone would have been a simple shape to use for your estimation.

Total = 36000 + 1000 = 37000mm^3 ✓

(interpretation)

b) True Volume

→ **Sphere**

VE = 4/3 x π x 20 x 20 x 20 = 33510.3 mm^3 ✓

→ **Octagonal Prism & Semi Sphere**

Octagonal Prism

VE = 2 (1+ √2) x 10^2 x 30 = 14485.3 mm^3 ✓

Semi Sphere

33510.3

VE = 4/3 x π x 20 x 20 x 20 = 32000 ÷ 2 = 16755.2 mm^3 ✓

Total Volume = 14485.3 + 16755.2 = 31240.5 mm^3 ✓

(interpretation)

Showing formulas and explaining what you are doing would have made it clearer.

→ **Cylinder & Hexagonal Pyramid**

Cylinder

VE = π x 20^2 x 30 = 37699.1 mm^3

Hexagonal Pyramid

VE = $\sqrt{3}/2 \times 10^2 \times 20 = 1732\ mm^3$

Total Volume = $37699.1 + 1732 = 39431.1 mm^3$ ✓

c) Comparison & Wastage

Explanation needed

Absolute Error = $|VA - VE|$

$$\text{Percentage Error} = \frac{|VA - VE|}{VE} \times 100\%$$

→Sphere

Absolute Error = $32000 - 33510.3 = |-1510.3| =$ **$1510.3\ mm^3$**

$$\text{Percentage Error} = \frac{|32000 - 33510.3|}{33510.3} \times 100\% = \mathbf{4.5\%}$$ ✓

→Octagonal Prism & Semi Sphere

Absolute Error = $28000 - 31240.5 = |-3240.5| =$ **$3240.5\ mm^3$**

$$\text{Percentage Error} = \frac{|28000 - 31240.5|}{31240.5} \times 100\% = \mathbf{10.3\%}$$ ✓

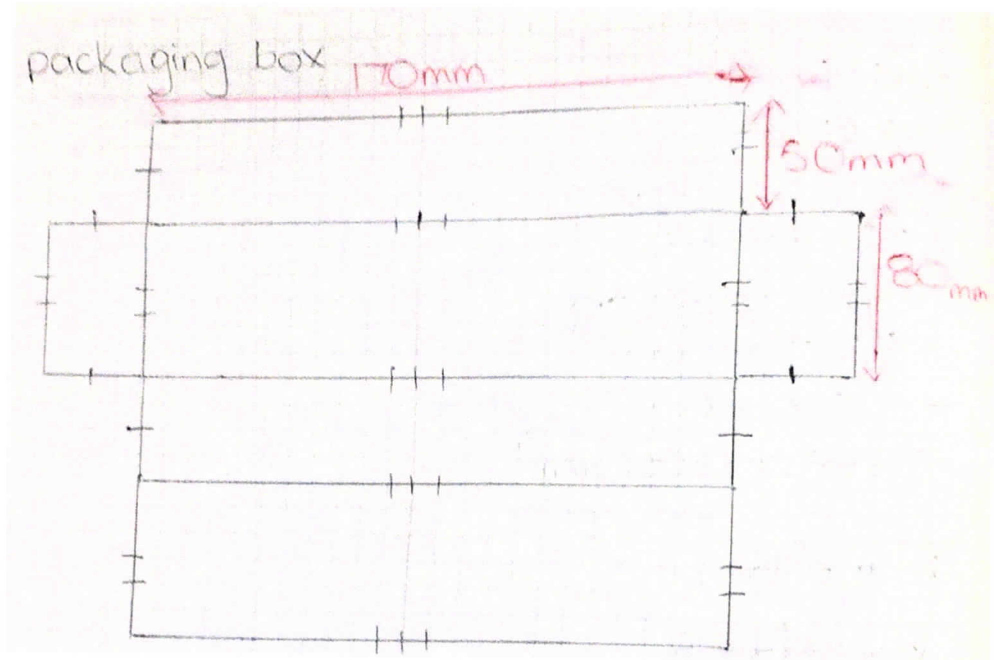
→Cylinder & Hexagonal Pyramid

Absolute Error = $37000 - 39413.1 = |-2413.1| =$ **$2413.1\ mm^3$**

$$\text{Percentage Error} = \frac{|37000 - 39413.1|}{39413.1} \times 100\% = \mathbf{6.1\%}$$ ✓

Above, absolute and percentage error are shown for the volume calculations for the sphere. As shown in the calculations above, the estimation was slightly inaccurate. A more accurate prediction could have been made through manipulating other formulas. Wastage is a detriment to a business, costs could be reduced by constructing pieces which are standard and can be efficiently stored and minimizing spare room in the box, resulting in cost-efficient pieces.

Part Three – Calculating Surface Area



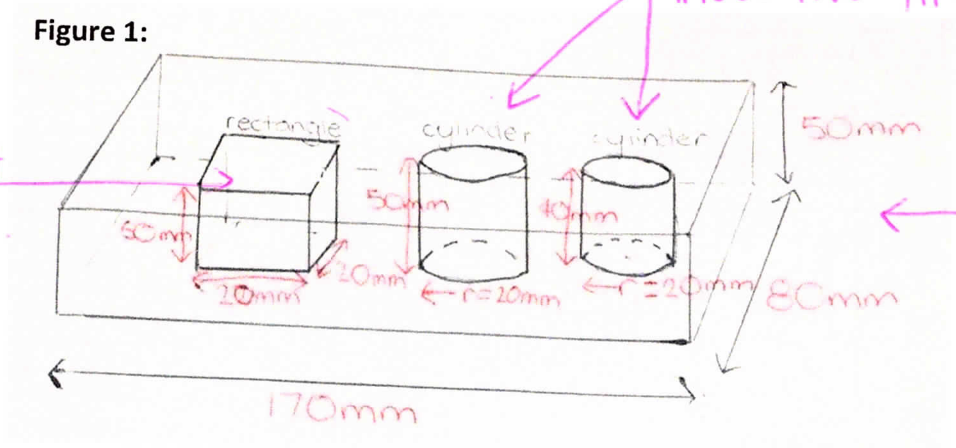
Above is the packaging box, a net diagram has been constructed to find the surface area. The following calculations show my method of working out the surface area of the packaging box:

170 x 50 = 8500mm^2
170 x 80 = 13600mm^2
50 x 80 = 4000mm^2

Total SA = 2(8500) + 2(13600) + 2(4000) = 52200mm^2

these two fit shape 1 & 3

Figure 1:



this doesn't fit shape 2.

Would the plastic go the whole height of the box? Would that make getting the chocolate out easy?

Figure 1 depicts the plastic tray which will hold and protect the three unique chocolate pieces inside the box. A rectangle and 2 cylinders with unique measurements have been used to keep the three pieces of chocolate in. height

$2a^2 = 10^2$
$a^2 = \frac{100}{2}$
$a^2 = 50$
$\therefore a = \sqrt{50}$
$= 7.07$

$\therefore$ Width = 7.07 x 2 + 10 = 24.14mm

Figure 2:

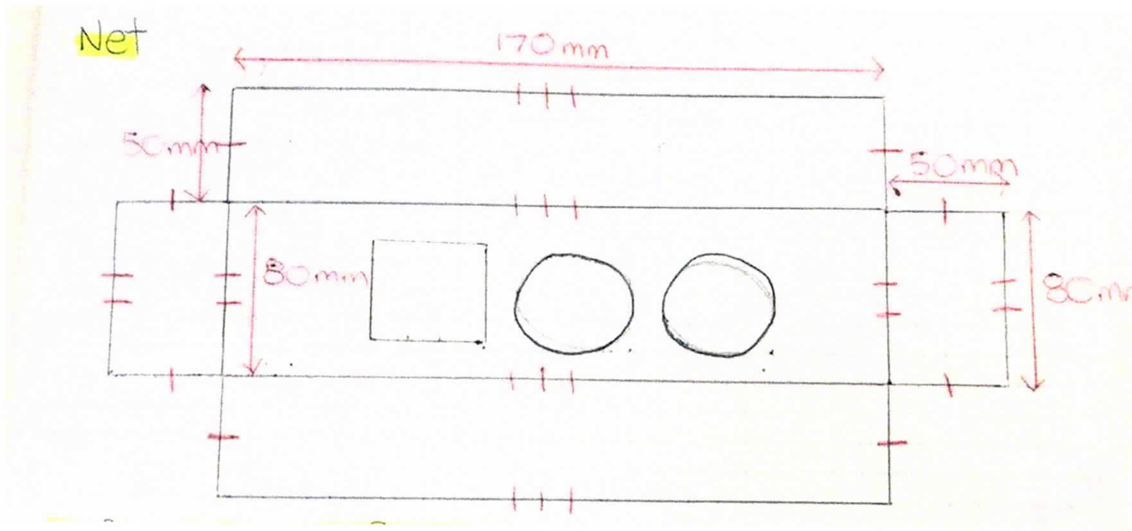


Figure 2 depicts the net diagram of the plastic tray. Dimensions of 170mm (length) x 80mm(width) x 50mm (height) have been used. The following calculations show my method of finding the total surface area of the tray:

170 x 50 = 8500mm^2
170 x 80 = 13600mm^2
50 x 80 = 4000mm^2

13600 + 2(4000) + 2(8500) = 38600mm^2 ✓

You should explain that you don't calculate bottom as you already have above

Cylinder (without top and bottom): 2 x π x 20 x 50 = 6283.2mm^2

Cylinder (without top and bottom): 2 x π x 20 x 40 = 5026.5mm^2

Rectangle: 2(20x20+50x20+50x20) = 4800 – 20 x 20 – 20 x 20 = 4000mm^2

↳ a simpler way is 20x50x4 = 4000mm^2

Total Surface Area = 38600 + 6283.2 + 5026.5 + 4000 = 53909.7mm^2 ✓

Part Four – Calculating Cost

Chocolate Cost = $20.50 for a 1kg bag

Piece 1 - Sphere → Volume = 33510.3mm^3

33510.3 ÷ 1000 = 33.5103cm^3

1cm^3 = 1 gram ✓

33.5103cm^3 = 33.5g

(g to kg)
33.5 ÷ 1000 = 0.033kg

0.033 x 20.50 = $0.68 ✓

Piece 2 – Octagonal Prism & Semi Sphere → Total Volume = 14485.3 + 16755.2 = 31240.5 mm^3

31240.5 ÷ 1000 = 31.23cm^3

1cm^3 = 1 gram

31.23 cm^3 = 31.23g

(g to kg)
31.23 ÷ 1000 = 0.031kg

0.031 x 20.50 = $0.63 ✓

Piece 2 – Cylinder & Hexagonal Pyramid → Total Volume = 37699.1 + 1732 = 39431.1 mm^3

39431.1 ÷ 1000 = 39.43cm^3

1cm^3 = 1gram

39.43cm^3 = 39.43g

(g to kg)
39.43 ÷ 1000 = 0.039kg

0.039 x 20.50 = $0.80 ✓

<u>Tray</u> = $4.30 per square meter

TSA of Tray = 53909.7 mm^2

53909.7 ÷ 1000000 = 0.0539097m^2 = 0.05m^2

4.30 x 0.05 = $0.22

<u>Packaging Box</u> = $2.80 per square meter ✓

TSA of Box = 52200mm^2

52200 ÷ 1000000 = 0.05m^2

2.80 x 0.05 = $0.14

Total Cost = 0.68 + 0.63 + 0.80 + 0.22 + 0.14 = $2.47 = $2.50 ✓

Explaining what you are doing and interpreting answers would have helped make it clearer.

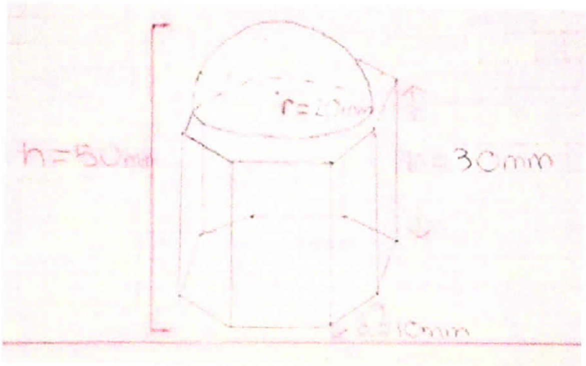
Part Five – The Report/Conclusion

In this investigation, measurement skills have been used to construct three uniquely designed pieces of chocolate, wherein, the volume will need to be found and detailed sketches need to be drawn. Estimations were made by manipulating other formulas for other shapes and replacing π with 3. Then true volume was calculated, followed by calculating absolute and percentage error. A plastic tray and packaging box are sketched out, including net diagrams which were used to find the surface area, lastly the costing is added up, overall achieving a $2.50 cost. ✓ Conclusion.

If a business were to construct these pieces I have created, it would not be cost-efficient as wastage has occurred, costs could be reduced by constructing pieces which are standard and can be efficiently stored and minimizing spare room in the box.

Throughout the investigation, various assumptions have been made. It was presumed that it was a solid piece of chocolate, not being hollow or filled with anything other than chocolate, ✓ the thickness of the tray was also assumed. The limitations in the investigation were the given dimensions, being 170mm (length) x 80mm (width) x 50mm (height), these restricted the chocolate piece sizes. ✓

→ what about the fact that you can't purchase just a small amount of material

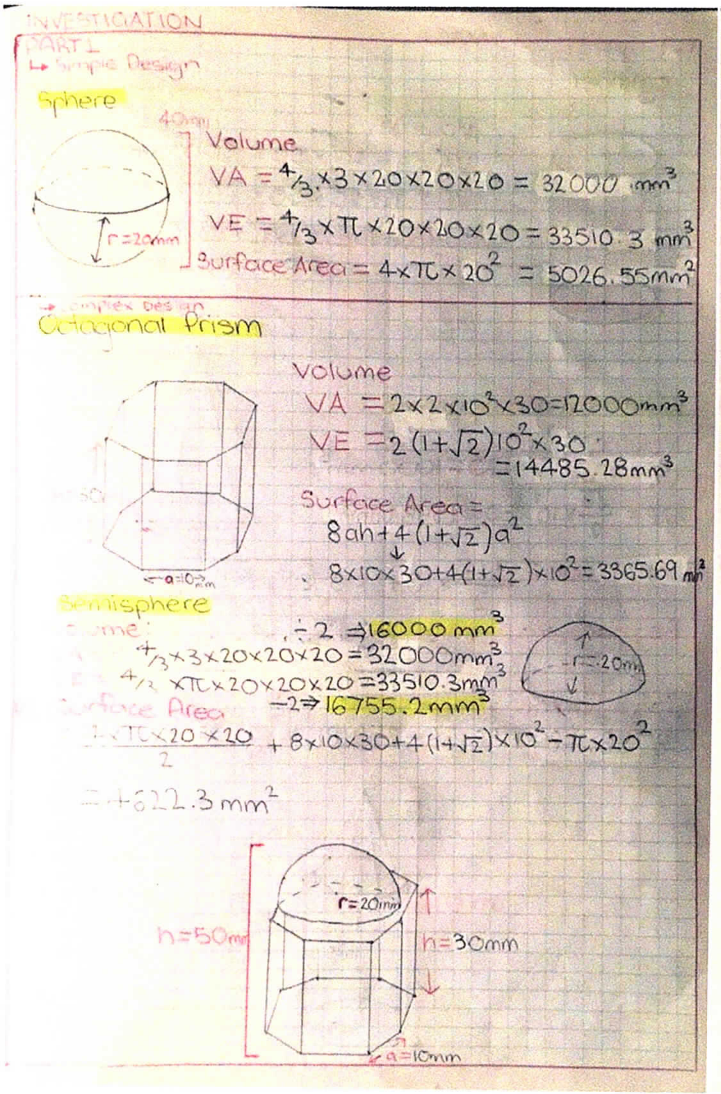


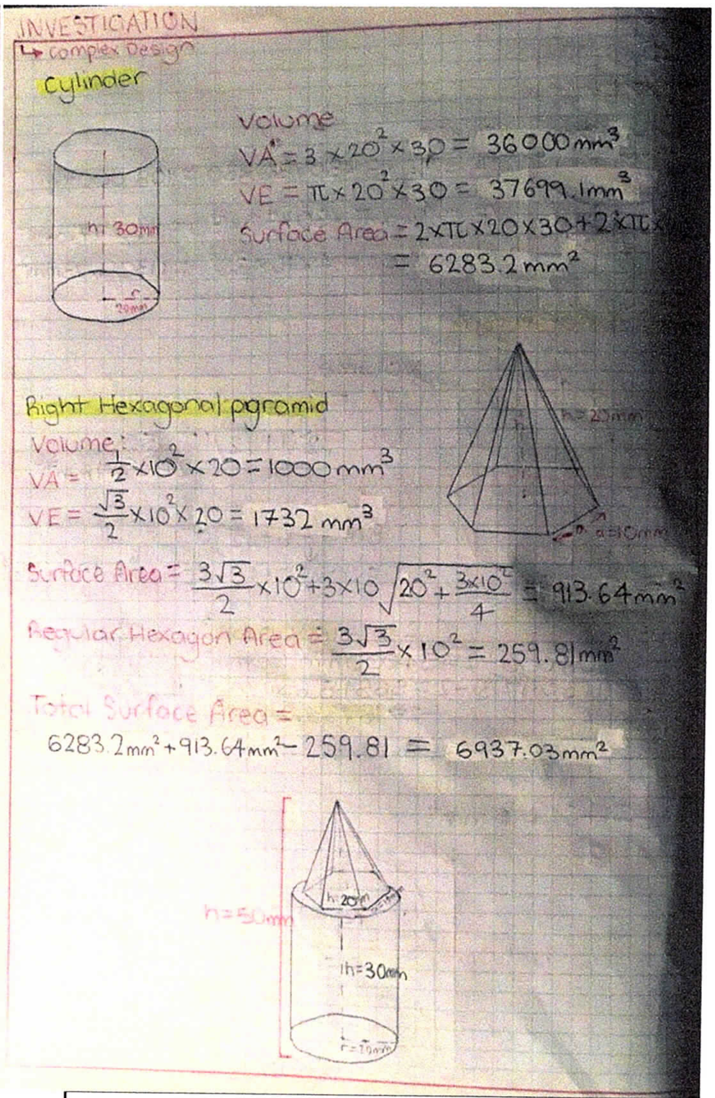
To the left is a scaled Diagram of one of the complex pieces I have designed, the octagonal prism and semi-sphere.

Through attentive calculations, detailed 3D sketches and various net diagrams, volume and surface area was found for each of the 3 pieces, then the packaging box and plastic tray, subsequently transferred to cm^2 in which the price was retrieved through converting it to grams then to kilograms, finally receiving a price of $2.50. Overall, the investigation conducted could be defective due to having ranging volumes and the shapes used for the tray could possibly cause the complex shapes not to fit in appropriately.

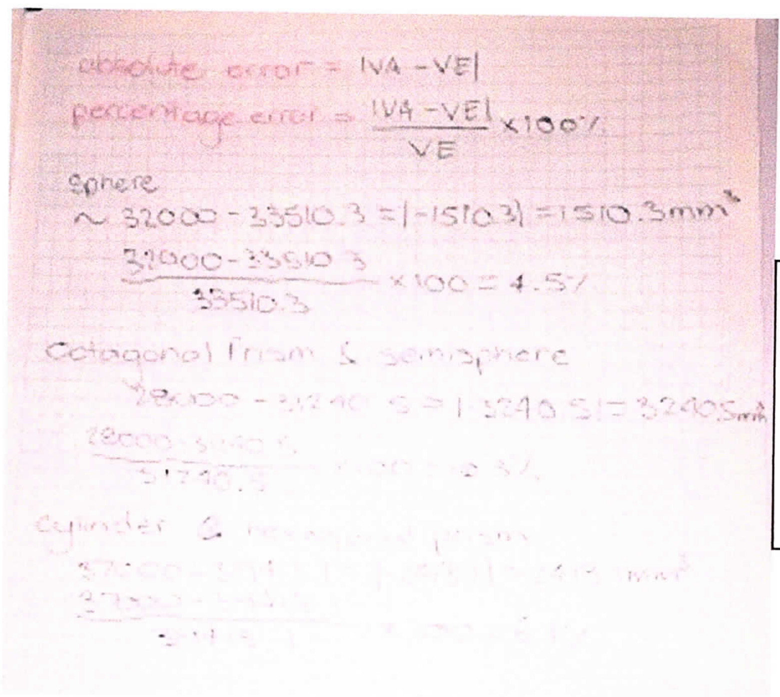
Main areas needing improvement were the interpretation of the answers in context, as well as communication of what you were doing. (RC1 & RC4)

Appendices



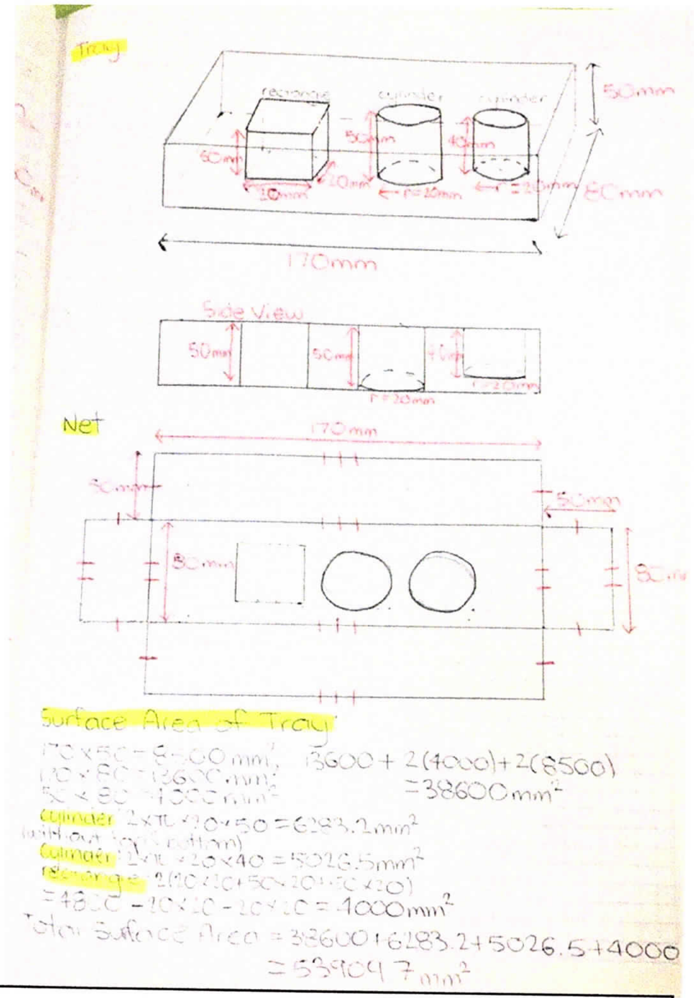


3D Sketches of each piece with the estimated volume and true volume. Surface area of each piece is also calculated.

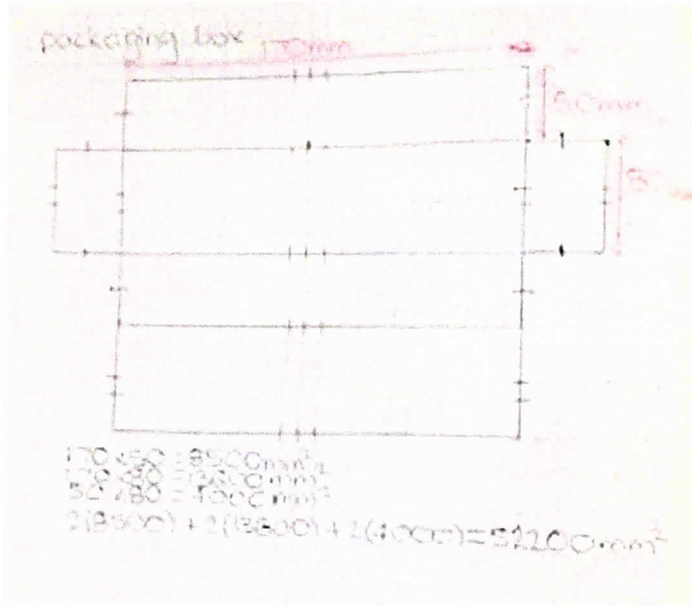


Calculating absolute and percentage area for the volume of each piece.

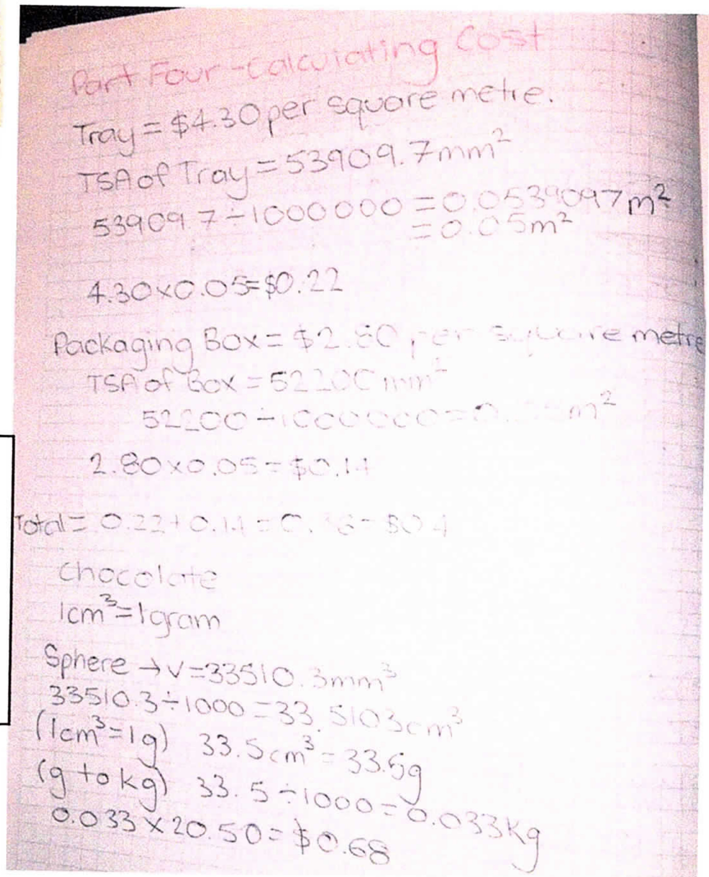
Appendices



Various Sketches, Net Diagram and total surface area of the plastic tray.



Net Diagram and total surface area of the packaging box.



Calculating the cost for the packaging box, chocolate and the tray.