# STAGE 1 MATHEMATICS

**Assessment Type 2: Mathematical Investigation**

**Introduction to Differential Calculus – Cake Tin**

A cake tin manufacturer will be making cake tins ranging in size from “tiny” to “gigantic”. Some tins will be square based, others will be rectangular based. In all cases the manufacturer wants to maximise the volume of each cake tin.

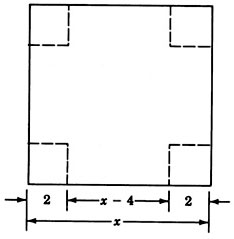
**Task**

This task investigates the size of a square cut into a piece of tinplate to form an open top cake tin which optimises the volume. Through the use of various mathematical calculations involving calculus and the quadratic formula, a conjecture will be made and potentially proven.

**Part A**

Investigate the square piece of tinplate.

An open top cake tin is to be made by cutting a square from each corner of a square piece of tinplate with side lengths . Once the cut is made the sides are folded to form an open top cake tin. Let cm be the side length of the square cuts to be made.



*cm*

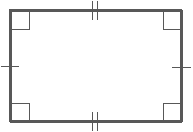
1. Given the length of each side of the tinplate is 5cm, show that the volume of the cake tin can be expressed as cm3

Hence, determine and, using the quadratic formula, find the exact value of which will maximise the volume of the cake tin.

1. Further this investigation by determining the exact value of for ***at least three*** other values of, the side lengths of the original square tinplate.
2. Present a conjecture based on a square piece of tinplate of side length , which when a square is cut from each corner of length a maximum volume for the resulting open top cake tin will be obtained.

**Part B**

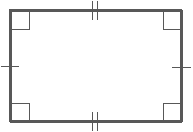
Consider the following rectangular piece of tinplate. An open top cake tin is to be made by cutting a square from each corner.

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Develop a conjecture about the relationship between (the cut to be made for the square) and the length of each side of the rectangle such that the cake tin has a maximum volume.

The sides of the rectangle are in a ratio . Consider a rectangle where one side is twice the length of the other (i.e. 2:1). Find the value of that gives the maximum volume for the cake tin. Repeat this process for rectangular tinplates with sides in at least two other ratios.

Hint: find exact solutions for (i.e. use the quadratic formula).



**Further extension**

Prove, using the principles of calculus (and the quadratic formula), one or both of the conjectures you have found from Part A and/or Part B.

**Notes**

Your report on the mathematical investigation should include the following:

* an outline of the problem and context
* the method required to find a solution, in terms of the mathematical model or strategy used
* the application of the mathematical model or strategy, including:
  + relevant data and/or information
  + mathematical calculations and results, using appropriate representations
  + the analysis and interpretation of results, including consideration of the reasonableness and limitations of the results
* the results and conclusions in the context of the problem
* a bibliography and appendices, as appropriate.

The format of an investigation report may be written or multimodal.

The investigation report should be a **maximum of 8 pages** if written, or the equivalent in multimodal form.

Performance Standards for Stage 1 Mathematics

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|  | **Concepts and Techniques** | **Reasoning and Communication** | |
| **A** | Comprehensive knowledge and understanding of concepts and relationships.  Highly effective selection and application of mathematical techniques and algorithms to find efficient and accurate solutions to routine and complex problems in a variety of contexts.  Successful development and application of mathematical models to find concise and accurate solutions.  Appropriate and effective use of electronic technology to find accurate solutions to routine and complex problems. | | Comprehensive interpretation of mathematical results in the context of the problem.  Drawing logical conclusions from mathematical results, with a comprehensive understanding of their reasonableness and limitations.  Proficient and accurate use of appropriate mathematical notation, representations, and terminology.  Highly effective communication of mathematical ideas and reasoning to develop logical and concise arguments.  Effective development and testing of valid conjectures. |
| **B** | Some depth of knowledge and understanding of concepts and relationships.  Mostly effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine and some complex problems in a variety of contexts.  Some development and successful application of mathematical models to find mostly accurate solutions.  Mostly appropriate and effective use of electronic technology to find mostly accurate solutions to routine and some complex problems. | | Mostly appropriate interpretation of mathematical results in the context of the problem.  Drawing mostly logical conclusions from mathematical results, with some depth of understanding of their reasonableness and limitations.  Mostly accurate use of appropriate mathematical notation, representations, and terminology.  Mostly effective communication of mathematical ideas and reasoning to develop mostly logical arguments.  Mostly effective development and testing of valid conjectures. |
| **C** | Generally competent knowledge and understanding of concepts and relationships.  Generally effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine problems in a variety of contexts.  Successful application of mathematical models to find generally accurate solutions.  Generally appropriate and effective use of electronic technology to find mostly accurate solutions to routine problems. | | Generally appropriate interpretation of mathematical results in the context of the problem.  Drawing some logical conclusions from mathematical results, with some understanding of their reasonableness and limitations.  Generally appropriate use of mathematical notation, representations, and terminology, with reasonable accuracy.  Generally effective communication of mathematical ideas and reasoning to develop some logical arguments.  Development and testing of generally valid conjectures. |
| **D** | Basic knowledge and some understanding of concepts and relationships.  Some selection and application of mathematical techniques and algorithms to find some accurate solutions to routine problems in some contexts.  Some application of mathematical models to find some accurate or partially accurate solutions.  Some appropriate use of electronic technology to find some accurate solutions to routine problems. | | Some interpretation of mathematical results.  Drawing some conclusions from mathematical results, with some awareness of their reasonableness or limitations.  Some appropriate use of mathematical notation, representations, and terminology, with some accuracy.  Some communication of mathematical ideas, with attempted reasoning and/or arguments.  Attempted development or testing of a reasonable conjecture. |
| **E** | Limited knowledge or understanding of concepts and relationships.  Attempted selection and limited application of mathematical techniques or algorithms, with limited accuracy in solving routine problems.  Attempted application of mathematical models, with limited accuracy.  Attempted use of electronic technology, with limited accuracy in solving routine problems. | | Limited interpretation of mathematical results.  Limited understanding of the meaning of mathematical results, their reasonableness or limitations.  Limited use of appropriate mathematical notation, representations, or terminology, with limited accuracy.  Attempted communication of mathematical ideas, with limited reasoning.  Limited attempt to develop or test a conjecture. |