2022 Mathematical Methods Subject Assessment Advice

Overview

Subject assessment advice, based on the 2022 assessment cycle, gives an overview of how students performed in their school and external assessments in relation to the learning requirements, assessment design criteria, and performance standards set out in the relevant subject outline. They provide information and advice regarding the assessment types, the application of the performance standards in school and external assessments, and the quality of student performance.

Teachers should refer to the subject outline for specifications on content and learning requirements, and to the subject operational information for operational matters and key dates.

School Assessment

Teachers can improve the moderation process and the online process by:

* thoroughly checking that all grades entered in school online are correct and aligned with the selected performance standards in the PSR
* ensuring the uploaded tasks are legible, all facing up (and all the same way), and remove blank pages, student notes and formula pages
* ensuring the uploaded responses have pages the same size and in colour so teacher marking and comments are clear
* for SATs and Mathematical Investigations responses, clearly marked answers showing which mathematical calculations are fully or partially correct and which are incorrect is a requirement of moderation. Showing marks and totals for SATs is also helpful
* uploading the SATs as a single scanned file
* preferably providing a summary of student results in each of the SATs on the first page of the uploaded SAT’s file
* using the same tasks when combining with another school or schools where possible to support comparable assessments. When combining classes across schools, teachers should be involved in moderation activities prior to uploading materials for the actual moderation to ensure that the rank order of the students within the combined assessment group is appropriate.

Assessment Type 1: Skills and Application Tasks (50%)

Students complete six skills and application tasks under the direct supervision of the teacher. The equivalent of one skills and applications task must be undertaken without the use of either a calculator or notes.

This year teachers were able to reduce one SAT through using the Covid adjustments offered in the subject. It is important to remember that where this has been applied, it should have been the same task for all students in the assessment group.

Students provide evidence of their learning in relation to the following assessment design criteria:

* concepts and techniques
* reasoning and communication.

Teachers can elicit more successful responses by:

* including tasks that provided a mix of routine and more complex problems that effectively differentiated student mathematical knowledge and understanding of concepts and relationships across the grade bands
* allowing multiple opportunities to demonstrate conjecture development and proof
* including several questions that allow students to demonstrate their interpretation of concepts and results
* including tasks that provided the opportunity for interpretation of the mathematical results in the context of the problem
* including questions that allowed for multiple entry points into extended questions by including some ‘show that‘ parts to enable students to access the following sections of the question even if they do not manage to calculate the correct answer in the ‘show that’ part
* providing students with axes and grids when asking them to sketch graphs.

The more successful responses commonly:

* provided detailed evidence of the development and proof of one or more conjectures
* consistently demonstrated mathematical knowledge and understanding of concepts and relationships across all topics
* showed all algebraic working by providing relevant steps, particularly for the ‘show that’ type questions
* used correct notation and terminology.

The less successful responses commonly:

* were seen in tasks that did not provide opportunity for students to solve more complex problems
* did not demonstrate an appropriate balance of understanding the key questions and concepts across the topics in the subject outline
* did not provide enough attention to detail in presenting their solutions
* lacked an understanding of how technology could be used effectively
* did not often attempt questions that asked for interpretation or logical conclusions of results
* were based on tasks that used questions directly from a textbook or from past exams
* were based on tasks that provided questions that were repetitive which either disadvantaged students with poor knowledge and understanding, or advantaged students with highly effective knowledge and understanding.

Assessment Type 2: Investigation (20%)

Students complete one mathematical investigation with minimal teacher direction. The task must afford students the opportunity to extend the investigation in an open-ended context. The task should be written in a format that allows the student to conduct their own open-ended investigation. It must be completed in report format (if written) and must be no longer than 15 single-sided A4 pages. Appendices should be used for repetitive calculations only.

Students provide evidence of their learning in relation to the following assessment design criteria:

* concepts and techniques
* reasoning and communication.

Teachers can elicit more successful responses by:

* providing some structure of an initial problem leading to a more open-ended problem to investigate, where the student can show the development and application of mathematical models through individual choices, refinements/improvements with justification for their rationale
* ensuring that the investigation is at an appropriate level of complexity and aligns well with the subject outline
* ensuring that the design of the task allows for the discussion of limitations and reasonableness. This is best done by providing a task in the context of a situation
* avoiding the use of question-and-answer style investigations, which limit student success ensuring the design of the task allows for and explicitly encourages the discussion of limitations and reasonableness of the modelling process:
* tasks that are designed to look at the generation of curves or shapes by altering values within formulae is not likely to result in individual work that is sufficiently open ended or allowing deep discussions concerning the reasonableness of solutions or limitations encountered
* presenting the response in report format. Communication of mathematical information is best done by using appropriate headings, labelling graphs and tables, referring to them in the main body of the response, and using appendices for repetitive calculations.

The more successful responses commonly:

* included detailed development and application of a mathematical model
* require mathematics beyond just modelling an equation using technology for a set of data
* demonstrated a comprehensive understanding of the reasonableness and limitations of the model developed, which can more easily be done in the context of a situation
* effectively communicated mathematical reasoning through the use of appropriate and well-labelled graphs and the inclusion of relevant calculations and notation
* made appropriate use of the appendices to avoid repetition in the report
* were based on tasks that included an element of student directed learning that differentiated student understanding of the model or problem being investigated
* effectively and efficiently used technology to solve problems.

The less successful responses commonly:

* included excessive scaffolding with routine and often repetitive calculations that were prescribed in the task design
* had limited supporting evidence of how the models were derived (e.g. trial and error, Desmos, researched and adapted)
* did not effectively select and apply appropriate mathematical techniques or use appropriate electronic technology when solving problems
* lacked depth in the interpretation of the mathematical results, often summarising what was calculated rather than analysing the results in the context of the problem
* were difficult to follow, lacking in explanation of choices made in developing a model
* relied on proofs that are easily accessible through the textbook or online
* were based on tasks that had a limited open-ended component that students found difficult to further develop a mathematical model for beyond fitting a curve.

Operational Advice

If students present their responses in oral or multimodal form, 6 minutes is the equivalent of 1000 words. Students should not speed-up the recording of their videos excessively in an attempt to condense more content into the maximum time limit.

From 2023, if a video is flagged by moderators as impacted by speed, schools will be requested to provide a transcript and moderators will be advised to moderate based on the evidence in the transcript, only considering evidence up to the maximum word limit.

If the speed of the recording makes the speech incomprehensible, it affects the accuracy of transcriptions and it also impacts the ability of moderators to find evidence of student achievement against the performance standards.

External Assessment

Assessment Type 3: Examination

In general, data suggests that students felt the examination was approachable and they were able to demonstrate their knowledge of the curriculum well. Evidence suggests that students more successfully managed their time this year than in the past, with most students being able to at least attempt all questions. Both the examination markers and I were pleasantly surprised by students’ general ability to apply their knowledge in unfamiliar contexts and successfully solve questions of considerable complexity.

The markers of the examination go to great effort to ensure students are awarded marks for evidence of understanding in responding to questions wherever possible; however, the following dot points are given below to allow students to achieve improved results overall more consistently. It’s worth noting that several of the dot points listed below have been listed previously in my feedback for past examinations; however, I have chosen to list them again as the examination markers and I continue to unfortunately see students not gain marks that they were seemingly capable of.

When completing their examination, students should:

* not cross out their responses or attempted responses to questions in the examination booklet unless they are confident that no part of their response should be considered by the marker
* clearly let markers know if they complete a question on one of the blank pages available to ensure that it is considered as a possible response to a question
* ensure that their answer makes sense in the context of the problem. Although one trivial error within a question is not penalised, if an unrealistic answer is obtained through error, a student should write a comment addressing this
* pay closer attention to the wording of questions. Words or phrases such as ‘exact’, ‘hence’, ‘show’ or ‘using an algebraic process’ are used to help guide a student’s approach in finding a solution
* take greater note of the allocated marks given in a question (and the space provided) to determine if they can use technology or simply state the answer (rather than show any mathematical process). Unless a particular approach is suggested by the wording of the question, students should always use the optimal method to find a solution
* understand that even if they do not successfully solve one part of a question, they can generally still continue to attempt to solve the following sections. Great care is taken during the writing process to allow multiple entry points into questions wherever possible
* be more careful when rounding numbers appropriately. It is an expectation of the course that answers are given to 3 significant figures (unless otherwise stated). One example was an answer of 0.0205 (correct to 3 significant figures). Many students listed this as 0.020 (only correct to 2 significant figures) or 0.02 (only correct to 1 significant figure). In order to not unfairly penalise a student multiple times throughout both booklets, we often pick a specific question to penalise rounding, and luckily for many students this year, this was not that specific question. However, although a mark was not taken off on this occasion, in a different exam, students could have easily not have gained a mark for these truncated responses
* improve their understanding of sign diagrams and use them more often to justify the nature of critical points.

Question 1

This question, much like many recent papers had the intention of allowing students to demonstrate routine calculus skills; however, the regular structure was altered slightly to include a very simplistic first principles question to begin the exam. Overall students were very successful in this question, with approximately 60% of students achieving 10 or 11 out of the 11 marks on offer. However, data suggests that many students continue to find it challenging to successfully implement a fully correct and thorough process to find a derivative using first principles.

The more successful responses commonly:

* used limit notation correctly throughout in part (a). This involves including when *h* was present and removing it when letting *h* tend towards zero
* correctly implemented the chain rule and quotient rule when required in part (b)
* used the quotient rule in part (b)(ii) instead of rearranging the equation to become a product rule.

The less successful responses commonly:

* did not correctly substitute into (or equivalent) in part (a)
* did not show clear simplification of the numerator and cancellation of *h* in part (a). Students have the responsibility in first principles questions to demonstrate additional steps to allow the marker to discern that the answer was not obtained through simple derivative laws
* expanded as in part (a)
* did not use brackets when simplifying the numerator leading to a common algebraic error in part (a)
* did not include appropriate brackets in their answers to part (b) and (c)
* incorrectly treated part (b)(i) as a product rule (i.e. )
* forgot to include the constant (i.e.) in their answer to part (c).

Question 2

This question contained some routine skills involving discrete random variables, in addition to some more complicated interpretation of a confidence intervals in its final part. Students generally were able to demonstrate their knowledge, skills and understanding well, evident in this question having the second highest mean percentage of marks gained (74.1%) to Question 1.

The more successful responses commonly:

* concisely described that the probabilities in the table were ‘not equal’ in their answer to part (a)
* used technology to find their confidence interval for the mean in part (d)(i), rather than substituting values into the formula manually
* used concise wording in part (d) (ii) and referred to the number 3.441. Students who achieved all marks in this question clearly stated that 3.441 was below the lower limit of the confidence interval (or conversely that the confidence interval was above 3.441).

The less successful responses commonly:

* misinterpreted part (a) as ‘why is this not a valid probability distribution?’, leading to incorrect answers such as ‘the probabilities do not sum to one’ (despite the fact that they, of course, do)
* interpreted part (b) as rather than the correct interpretation of
* used incorrect notation in their confidence interval, such as instead of the required in part (d)(i)
* had vague statements about the placement of 3.441 in relation to their confidence interval, such as ‘3.441 is outside the confidence interval’ which is not enough evidence to justify further investigation in part (d)(ii)
* referred to the ‘claim’ being supported or rejected without answering the question of whether it suggested that further investigation should be undertaken in part (d)(ii).

Question 3

This question asked students to interpret the shape of the given graph to construct sign diagrams and a graph of its derivative. In order to make the question less routine, the graph contained turning points, inflection points and an asymptote. The data suggest that students found this question challenging, with an average of 4.15 achieved out of 7 marks possible.

The more successful responses commonly:

* included the vertical asymptote on their sign diagrams in part (a) and their graph in part (b)
* had a local minimum at in their graph in part (b)
* for the section of the drawn derivative graph in part (b) where the graph of the derivative had a negative slope and was above the *x*-axes.

The less successful responses commonly:

* included incorrect additional values in their sign diagrams in part (a). For example:
* in part (a) (i), students incorrectly included
* in part (a) (ii), students incorrectly included and
* did not denote the asymptote at on either of the sign diagrams in part (a)
* did not draw the graph for values of in part (b).

Question 4

This question was centred around knowledge of the Normal Distribution. It also assessed whether students could discern if a sample sum (in this case, ) was normally distributed using the Central Limit Theorem. Student use of technology was a focus here, and they generally achieved well in this question with approximately one quarter of students achieving full marks.

The more successful responses commonly:

* correctly used technology to quickly find answers to part (a), part (b)(iii) and part (c)
* understood the link between the value calculated in part (c) to quickly find the answer to part (d)
* expressed the equation in part (d) with on the left-hand size of the equation, avoiding a possible algebraic error.

The less successful responses commonly:

* incorrectly stated that a sample size of 6 is sufficiently large for the Central Limit Theorem to apply in part (b)(i). It’s important to note that even if the sample size was larger, more evidence is required to conjecture that a distribution is in fact approximately normal (for example, in past examinations, graphs with an approximate bell shape have been given)
* incorrectly considered *X* and to be a discrete random variable, resulting in the changing of limits of the probability statement throughout the question. For example, incorrectly becoming
* found a value of *k* such that in part (c), perhaps confusing the given inequality with the *z*-scores used in a 99% confidence interval
* used a trial an error approach in part (d) (often with insufficient accuracy, (i.e. not to 3 significant figures), not addressing the need to use the answer to part (c) outlined in the question).

Question 5

This question assessed student’s ability to use an algebraic process to find stationary points. The question was designed to involve a more challenging function, with a quadratic function used as the argument of a trigonometric function. Overall, students did particularly well in this question, and were able to correctly differentiate and solve for stationary points. In the final two parts of the question, students had to correctly use their answers to interpret properties of the graph of the function. This question was well completed by students, with the third highest average percentage of marks gained (71.6%) in booklet one.

The more successful responses commonly:

* showed clear steps when solving in part (b)(ii). Without clear working out, it was deemed that a student had used technology to find the requested value. Students needed to state that and hence, (with or without consideration of the periodic nature of the function due to the turning point having the smallest positive *x*-value)
* used their answer to part (b)(ii) to obtain the answer to part (b)(iii) with correct notation.

The less successful responses commonly:

* attempted to use to find *a* in part (b)(ii), ignoring the command term ‘hence’ used to direct students
* listed *a* as a truncated decimal, despite the word ‘exact’ used in the question in part (b)(iii).

Question 6

This question served to bring many parts of the curriculum together into a more challenging context. The question involved both a complicated derivative and optimisation question, in conjunction with both estimation of areas using rectangles and exact areas using technology. It also was intended to have students demonstrate understanding of the relationship between the ‘rate in’ and ‘rate out’ of a closed system.

The question was also written to address that the Course Outline now states that underestimates and overestimates can be found for non-monotonic functions, hence we deemed it appropriate to show an example of the rectangles used to calculate an estimation within the question.

Students found this question particularly challenging, with the lowest average percentage of marks gained in the first booklet of the examination (49.8%). Additionally, perhaps due to the large number of marks on offer, only a small number of students were able to get full marks in this question (3.34%), the lowest percentage across both booklets.

The more successful responses commonly:

* showed evidence of algebraic process and did not simply restate the given answer in part (a)(i). Examples of evidence that allowed students to achieve full marks were:
* brackets to demonstrate the use of the product rule
* being expressed as
* used a sign diagram in part (a)(ii) to demonstrate that it was a maximum at
* understood that the given maximum at in part (a)(ii) related to finding the overestimate in part (b)(i)
* concisely stated that the ‘rate of processing was greater than the rate of arrivals’ for the designated time in part (c)(i)
* factored in the given value of 600 (the initial number of samples present in the queue) into their answer to part (c)(iii)
* used information already calculated in the question (i.e. part (b)(ii)) to help simplify the process required to correctly find the answer to part (c)(iii).

The less successful responses commonly:

* attempted to use a rule relating to the product of three functions to demonstrate the answer given in part (a)(i) (i.e. ). Although the rule is correct, its complexity led to errors
* partitioned the function into and resulting in nested product rules in part (a)(i)
* did not consider the solution of in their solution to part (a)(ii) often as a result of incorrectly dividing both sides of the equation by (i.e. )
* did not supply evidence that it was a maximum at in part (a) (ii)
* did not use the correct *x*-values when calculating the heights of the rectangles in part (b)(i). Many students incorrectly used values such as for the height of the first rectangle
* did not state their value of *N* correct to the nearest integer as directed in part (b)(ii)
* assigned incorrect limits, or listed the difference between and in the incorrect order required to get a positive value of *N* in part (c)(ii)(1)
* had an answer to parts (c)(ii)(2) which was negative, without a comment addressing that this answer was inappropriate.

Question 7

Early parts of this question contained routine knowledge of Calculus, followed by a more challenging conjecture question requiring students to carefully implement laws of logarithms. Students were able to successfully solve the early parts of this question; however, many students continue to find it challenging to implement an appropriate process to successfully solve conjecture style problems. This is demonstrated in the data which showed that although a large number of students achieved full marks (21.5%), the overall average percentage marks gained was still rather low (56.1%). This lower average is also a result of a high percentage of students achieving 0 marks.

The more successful responses commonly:

* showed evidence of algebraic process, and did not simply restate the given answer in part (a)(i)
* used technology to quickly find the values for the table in part (b), expressing the values as fractions
* took note of the use of brackets (i.e. ) and realised that is a constant in the given equation at the top of page 4
* used part (a) to assist with their derivative in part (d)
* used laws of logarithms to simplify the original equation or the solution when setting in part (d). For example, .

The less successful responses commonly:

* did not show appropriate steps of logic in part (a)(ii). Without demonstrating these steps, it was deemed that the student used their graphics calculator to obtain the answer
* only found the *x*-coordinate of the stationary point throughout the question rather than the -coordinate
* found the value of the *y*-coordinate of the function at for instead of the *y*-coordinate of the local minimum in part (b)
* seemingly put in a random pattern of numbers into to the table in part (b), followed by a conjecture that matched this pattern
* formed a conjecture in part (c) that did not match the table of values in part (b)
* used a new value of *k* (such as ) in part (d) to show that their conjecture continues to work for this single value. It’s essential that students understand that this does not prove the conjecture for ‘any value of *k*’ as requested
* did not show clear evidence that when letting , the *y*-coordinate was in part (d). For example, many students reached an equation for the *y*-coordinate of , then, without evidence, stated that In order to achieve the allocated mark for this part of the question, they needed to show algebraic evidence that and cancel using laws of logarithms.

Question 8

This question combined many sections of the course such as discrete random variables, drawing of graphs using technology and inferential statistics. The conclusion of the question required students to consider the relationship between the limits of their calculated confidence interval and the given function of in an unfamiliar and difficult context. Data suggests that students found the final part of this question perhaps the most difficult question of the examination to successfully master with only 4.35% of students achieving full marks. However, the modal number of marks achieved in this question was high (12 marks out of 16 marks on offer), and it had the highest percentage of marks gained in the second booklet (60.7%).

The more successful responses commonly:

* correctly used technology to draw the graph in part (b). Some criteria expected of students were that:
* the end points at and should be approximately at the correct location
* the local minimum should be approximately at the correct location
* no inflection points to the left of the local minimum (ignoring any accidental ‘wobble’)
* the general shape of the graph should be correct with a clear inflection point to the right of the local minimum
* used technology to find their confidence interval for the proportion in part (e)(i), rather than substituting values into the formula manually
* clearly stated ‘no’ in their answers to part (e)(iii) in addition to the justification that the value of 0.0781 is inside the confidence interval
* could associate their calculated confidence interval with the visual aid of the graph of in Figure 8 to help guide their response to part (e)(iv)
* realised that the lower limit of the calculated confidence interval had no bearing on the requested upper and lower limit in part (e)(iv).

The less successful responses commonly:

* did not read part (a)(ii) carefully enough (or alternatively they considered the number of Choc-pricots to be a continuous random variable), leading to them incorrectly calculating ) rather than the required
* had their calculator set to measure angles in degrees rather than radians. Although efforts were made to not unfairly penalise students, this single mistake resulted in a graph in part (b) which was more challenging to draw, reduced the complexity of part (c), and it also made the answer to part (e)(iv) slightly inconsistent with the stated wording of the question
* only stated one of the two possible answers to part (c)
* attempted to use their graph drawn in Figure 8 to find answers to part (c) and part (d)
* used incorrect notation in their confidence interval, such as instead of the required *p* in part (e)(ii)
* answered a similar but not equitable question in part (e)(iii). Many students instead answered if the claim of was supported (or not) rather than if they could conclude if the claim is false. It seems than many students were using a pre-constructed response
* commented that the value of was within the confidence interval in part (e)(iii)
* treated the limits of the confidence interval as a rate of production rather than a probability that a randomly selected Choc-pricot was imperfect in part (e)(iv).

Question 9

This question contained no real-world context; however, the complexity of algebra required to solve the general equation of the normal resulted in it being a challenging question to achieve all marks on offer. It assessed students’ knowledge of the equation of a normal, and optimisation in the final section. Approximately half of students were able to achieve more than half the marks on offer, with a mean percentage of marks gains of 49.6%.

The more successful responses commonly:

* converted the given equation into index form (i.e. ) and then correctly applied the chain rule with evidence of process to obtain the given answer in part (a)
* implemented one of many possible systematic procedures to find the equation of the normal in part (b)(i) and part (c)(i) showing clear steps of working
* showed clear evidence of removing the factor of from the equation of the normal in part (c)(i)
* used knowledge of the properties of quadratic functions to determine the *x*-coordinate of the maximum in part (c)(ii)

The less successful responses commonly:

* found the equation of the tangent in part (b)(i), forgetting to use the negative reciprocal of the value of as the slope. This error was often followed by students illogically manipulating their equation to match the given answer
* did not show sufficient steps of working in part (b)(i) and part (c)(i)
* used a specific value of *a* (for example ) in part (c)(i) rather than consider the general case
* did not read the question carefully, setting , rather than setting  in part (c)(ii).

Question 10

This question introduced students to a new probability density function based on a slightly simplified Weibull distribution. Although the initial part of the question was routine, the following sections had students; interpreting their results, performing complicated unfamiliar derivatives, and solving unfamiliar exponential equations that finished with a solution involving nested natural logarithms. Overall, we were surprised by the number of students who were able to successfully complete the final part of this question with approximately 8% of students gaining all marks on offer. However, there continues to be a large number of students (21.8%) who did not achieve any marks, despite most at least attempting the question.

The more successful responses commonly:

* implemented technology to find an accurate solution to part (a)(i)
* referred to their answer to part (a)(i) (i.e. 0.430) and its position in relation to 0.5 when justifying their reasoning in part (a)(ii)
* showed clear evidence of process in part (b)(i). Some examples of evidence which helped students achieve full marks are:
* the presence of two negative signs in their working rather than combining them together to make a positive without evidence
* the use of appropriate brackets in their working
* listing *k* and as separate factors resulting from the derivative of the index of the exponential function
* were methodical in their approach to integrating the given function and expressing *k* as the subject of the equation in part (b)(ii). Students who were successful in this question were able to demonstrate:
* the linkage of part (b)(i) to the integral of the given function
* correct substitution of into the upper limit of the integral early in the algebraic process
* simplification of to 1
* correct use of logarithms. Students needed to ensure that both sides of the equation were positive before implementing logarithms on both occasions (often with different bases i.e. ) to make *k* the subject of the formula.

The less successful responses commonly:

* found in part (a)(i) rather than using the definite integral of 
* found *m* using technology in part (a)(ii), rather than referencing their answer to part (a)(i) as requested
* made algebraic errors in the process of attempting to isolate *k* in part (b)(ii). These errors include but are not limited to:
* equating the integral equal to 1.5 rather than 0.5
* simplifying to 0
* incorrectly taking logarithms of both sides of an equation when negative
* missing one logarithm in their final answer (i.e. )
* incorrectly reading  as  during the algebraic process, significantly reducing the complexity of algebra required to isolate *k*.