South Australian Certificate of Education

The purpose of this sample paper is to show the structure of the 130-minute Mathematical Methods examination and the style of questions that might be used. The examination will consist of questions that assess a *selection* of the key questions and key concepts from across the six topics.

Mathematical Methods
November 2020 sample paper

Question booklet 2

- Questions 7 to 10 (44 marks)
- Answer ***all*** questions
- Write your answers in this question booklet
- You may write on page 13 if you need more space
- Allow approximately 65 minutes
- Approved calculators may be used — complete the box below

2

Copy the information from your SACE label here

SEQ	FIGURES	CHECK LETTER	BIN

Graphics calculator

1. Brand _____

Model _____

2. Brand _____

Model _____

Government of South Australia

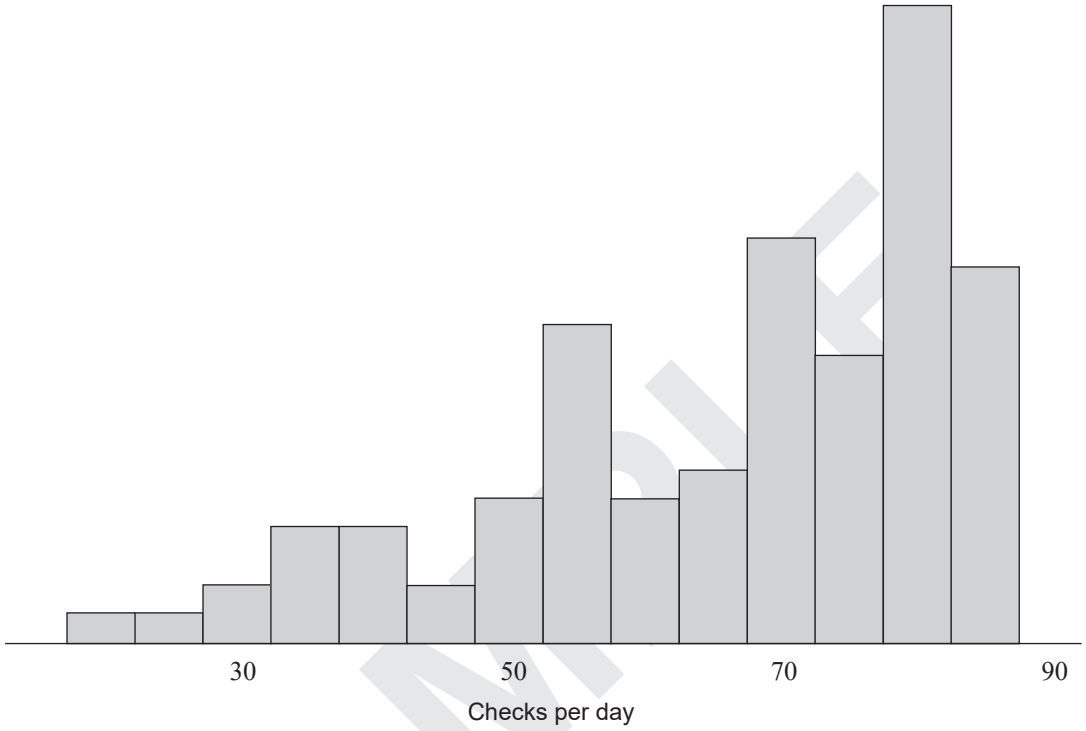
Question 7 (8 marks)

Concerns have been raised over the number of times per day that teenagers check their mobile device (such as a smartphone or a mobile / cell phone).

A study investigated the number of times per day that teenagers check their mobile device. A random sample of 100 teenagers was taken and the number of checks per day (X) was recorded.

The sample mean was found to be $\bar{x} = 62.33$. The sample standard deviation was found to be $s = 19.31$, which is assumed to represent a good estimate for the population standard deviation (σ_X).

The histogram below shows the number of checks per day for this sample of 100 teenagers.

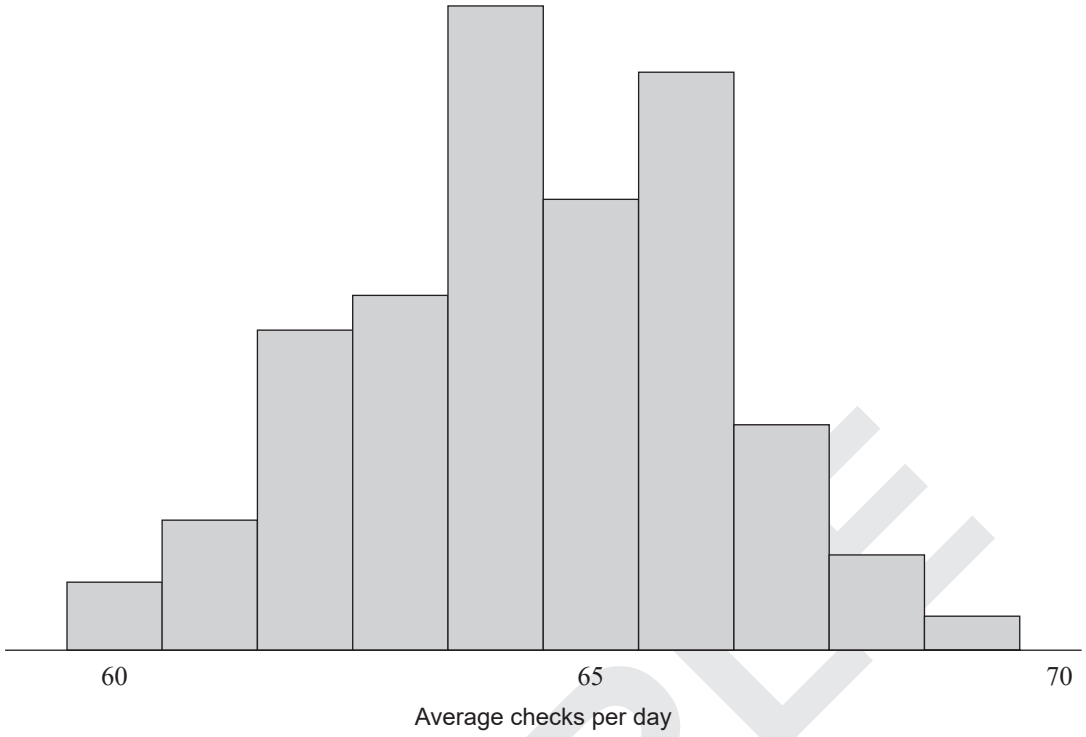


(a) With reference to the histogram above, explain why it may ***not*** be appropriate to use this sample to calculate a confidence interval for the population mean (μ_X).

(1 mark)

Suppose that many random samples of 100 teenagers were taken as part of the study, and the sample mean ($\bar{X}$) for each of these samples was recorded.

The histogram below shows the sample means.



(b) The central limit theorem applies when the sample size is 'sufficiently large'.

With reference to the histogram above, explain why, in this case, the sample size of 100 is sufficiently large.

(1 mark)

Question 7 continues on page 4.

(c) Using the sample of 100 teenagers considered in part (a):

(i) calculate a 90% confidence interval for the population mean (μ_X).

(2 marks)

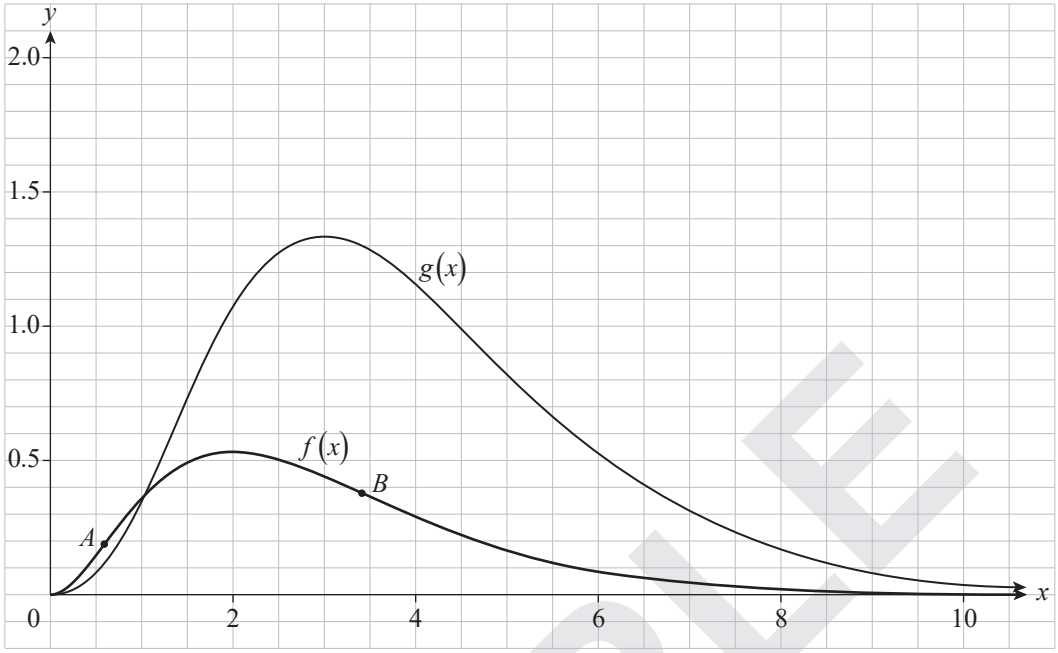
(ii) find the minimum sample size required in order to calculate a 90% confidence interval that has a width of no more than 5 checks per day.

(4 marks)

Question 8 (10 marks)

(a) Consider the functions $f(x) = e^{-x}x^2$ and $g(x) = e^{-x}x^3$.

The graphs of $y = f(x)$ and $y = g(x)$ for $x > 0$ are shown below.



Each graph has two non-stationary points of inflection for $x > 0$.

(i) On the graph of $y = g(x)$, clearly mark the location of the two non-stationary points of inflection. (2 marks)

(ii) Points A and B are the two non-stationary points of inflection on the graph of $y = f(x)$.

Find the coordinates of point A.

(2 marks)

Question 8 continues on page 6.

(b) Now consider the general function $h(x) = e^{-x}x^n$, for $x > 0$ and $n \geq 2$, where n is a real number.

(i) Show that $h''(x) = e^{-x}x^{n-2}\left(x^2 - 2nx + n(n-1)\right)$.

(3 marks)

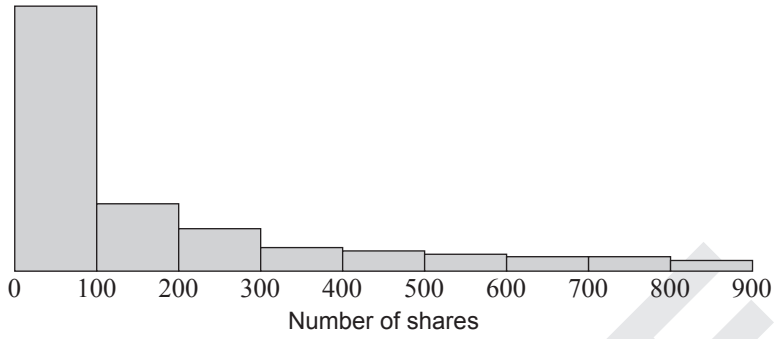
(ii) Hence show that the graph of $y = h(x)$ for $x > 0$ always has two distinct points of inflection.

(3 marks)

Question 9 (11 marks)

Some companies advertise their products by creating ‘memes’. They post these memes on social media in the hope that these will be ‘shared’ by a large number of social media users, thus promoting the product.

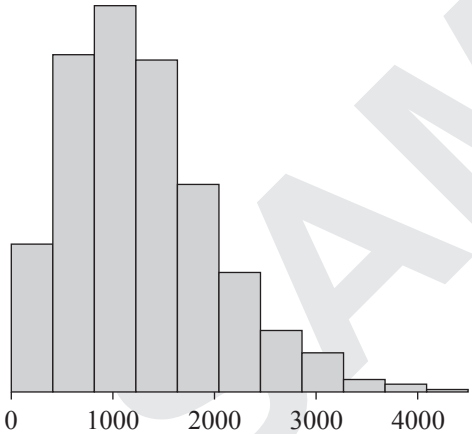
Scientists studied the number of ‘shares’ of memes for a particular product, called product C, and found that they follow the distribution depicted in the graph below. The distribution has a mean of 131.25 shares and a standard deviation of 221 shares.



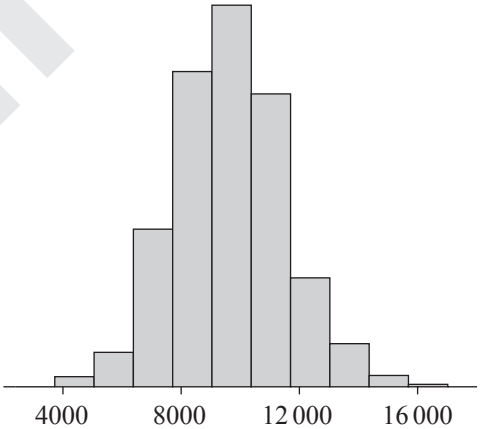
In an advertising campaign, companies may post a batch of memes on social media, in the hope that some of these will be shared by a large number of social media users. Let S_n represent the sum of the shares of a random sample of n product C memes.

(a) The two histograms below show the distributions of S_{10} and S_{75}.

Histogram A



Histogram B



Which histogram (**A** or **B**) corresponds to the distribution of S_{75}? Explain your answer.

(2 marks)

This sample Mathematical Methods paper shows the format of the examination from November 2020.

(b) Show that the distribution of S_{75} has a mean of 9844 shares and a standard deviation of 1914 shares.

(2 marks)

(c) Consider the distribution of S_n.

(i) As n becomes sufficiently large, what will the distribution of S_n tend towards?

(1 mark)

(ii) Is $n = 75$ sufficiently large in this case? Explain your answer.

(2 marks)

An advertising campaign is considered to be successful when the combined total number of shares of the memes in the batch is $10\,000$ or greater.

It can be assumed that the memes in an advertising campaign can be considered a random sample drawn from the population of all product C memes.

(d) What is the probability that an advertising campaign consisting of a batch of 75 product C memes will be successful?

(2 marks)

(e) A company would like their advertising campaign to have a probability of success that is greater than 0.8.

Show that this would be achieved by posting a batch of at least 90 product C memes on social media.

(2 marks)

Question 10 (15 marks)

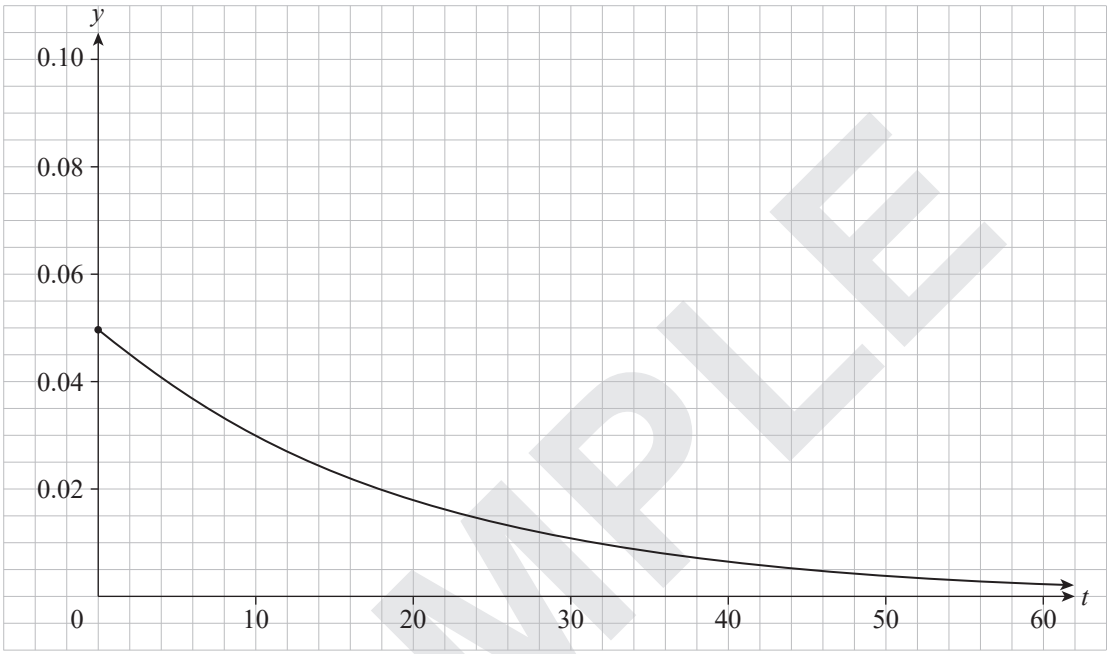
Source: © Tktktk | Dreamstime.com

At a sporting event attended by thousands of people, individuals spend time in a queue in order to enter the stadium. The probability that a randomly chosen individual spends time, t, in the entry queue can be modelled by the probability density function

$$f(t) = 0.05e^{-0.05t},$$

where t is measured in minutes and $t \geq 0$.

The graph of $y = f(t)$ is shown below.



(a) (i) Calculate the probability that a randomly chosen individual spends between 0 and 10 minutes in the entry queue.

(2 marks)

(ii) On the graph above, draw a representation of your answer to part (a)(i). (1 mark)

(b) (i) Calculate the probability that a randomly chosen individual spends *less than* 60 minutes in the entry queue.

(1 mark)

(ii) Of 200 individuals who entered the stadium, how many does the model predict spent *more than* 60 minutes in the entry queue?

(2 marks)

(c) Describe *one* limitation of using a function of the form $f(t) = a \times e^{-at}$ to model the time that an individual spends in the entry queue.

(1 mark)

It is a management policy that once inside the stadium, all individuals who spend time in a queue to buy food must be served within 20 minutes. The probability that an individual spends time t in the food queue can be modelled by the probability density function

$$g(t) = B \times e^{-0.05t},$$

where B is a positive value, t is measured in minutes, and $0 \leq t \leq 20$.

(d) Find the *exact* value of B.

(4 marks)

(e) On the axes on page 10, sketch the graph of $y = g(t)$. (2 marks)

(f) Is the probability of an individual spending between 0 and 10 minutes in the food queue greater than or less than the probability of an individual spending between 0 and 10 minutes in the entry queue?

Give a reason for your answer, *without* calculating the probability of spending between 0 and 10 minutes in the food queue.

(2 marks)

You may write on this page if you need more space to finish your answers to any of the questions in Question booklet 2. Make sure to label each answer carefully (e.g. 7(c)(ii) continued).