**STAGE 1 MATHEMATICS**

**PROGRAM 1 – SEMESTER 2**

This program is for a cohort of students intending to continue to Mathematical Methods at Stage 2. The following program describes the second semester of learning.

**SEMESTER TWO 17 WEEKS INCLUDING EXAM WEEKS AND STUDENT DEVELOPMENT/ACTIVITY WEEK**

* Topic 4 – Counting and Statistics (5 weeks)
* Topic 6 – Introduction to Differential Calculus (6 weeks)
* Topic 5 – Growth and Decay (4 weeks)

**Topic 4 – Counting and Statistics (5 weeks)**

| **Term Week** | **Subtopic** | **Concepts and Content**Technology is incorporated into all aspects of this topic as appropriate | **Assessment Task** |
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| WEEK 9 TERM 2 START SEMESTER TWO - STUDENT DEVELOPMENT/ACTIVITY WEEK  |
| 2-10 | 4.1Counting | The Multiplication Principle* The idea that if there are **a** ways of doing something and **b** ways of doing another thing, then there are **a** × **b** ways of performing both actions. Examples of tree diagrams, tables etc

Factorials and Factorial Notation* The factorial of a [non-negative integer](http://en.wikipedia.org/wiki/Non-negative_integer) n, denoted by n!, is the [product](http://en.wikipedia.org/wiki/Product_%28mathematics%29) of all positive integers less than or equal to *n*. For example, 4!=4×3×2×1=24

Permutations* Counting of all possible arrangements of a collection of things (discrete), where the order is important

 * Using only discrete variables, students explore various examples. Initially algebraically, then using technology.
 |  |
| 3-1 | Combinations* The number of ways to select different groups in which the order does not matter
* The number of combinations of objects taken from a set of distinct objects is
* Using only discrete variables, students explore various examples. Initially algebraically, then using technology.

Use of the notation * The coefficients of the expansion of
	+ Expand for integers
	+ Recognise the numbers as binomial coefficients
	+ The pattern connecting the values of leading to Pascal’s triangle
 |  |
| 3-2 | 4.2Discrete and Continuous Random Data | Definitions of and differences between discrete and continuous variables* Examples considered and students identify discrete and continuous data. Consideration given to continuous variables that may appear to be recorded as discrete.
 |  |
| 4.3Samples and Statistical Measures | Describing the centre of data: mean, mode and median* Students develop an understanding of the differences from calculations for each measure of centre. Consideration is given to strength and weakness of each e.g. how extreme value(s) may distort the mean.

Describing the spread of data: range, interquartile range and standard deviation* Students are aware of the standard deviation formula

The use of electronic technology to determine the above is implemented once the concepts are understood. |  |
| 3-3 | 4.4Normal Distributions | Normal distributions occur when the quantity is the combined effect of a number of random errorsFeatures of normal distributions* Bell-shaped
* Position of mean
* Symmetry about mean
* Characteristic spread
* Positions of one, two and three standard deviations from the mean. Technology for other values and inverse calculations if time permits. Use of the 68-95-99.7 rule.

Students use examples to understand the concepts and their implication in real life scenarios. |  |
| 3-4 |  | **Revision and SAT 1** | **SAT 1**1 hourEntire topicCalculator permitted  |

**Topic 6 – Introduction to Differential Calculus (6 weeks)**

| **Term Week** | **Subtopic** | **Concepts and Content**Technology is incorporated into all aspects of this topic as appropriate | **Assessment Task** |
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| 3-5 | 6.1Rate of Change | Discussion on rate of change as the ratio of the change in one quantity compared to another* Consider average speeds and other quantities
* Using graphical exemplars demonstrate how it is found that the average rate of change of a function on the interval from to is given by
* Connection between average rate of change and the slope of the chord
 |  |
| 3-6 | 6.2The Concept of a Derivative | How do we approximate the rate of change at a point (instantaneous rate of change)?* Technology will be implemented to demonstrate this concept. Students will use tables and formula to produce graphical representations to explore how as an interval from to decreases the approximation approaches the instantaneous rate of change.

Understanding what a limit is* The instantaneous rate of change of a function at a point is the limit of the average rate of change over an interval approaching zero. (The instantaneous rate of change at any particular point on a curve is the slope of the tangent to the curve drawn at the point.)

Understanding what a derivative is* Introduction to the concept that a derivative can be used to calculate the instantaneous rate of change

Introduction of first principles * Finding the derivative function from first principles
* Using first principles to find the derivative at a given point using
* Students use first principles to calculate derivatives of functions (only integer exponent)
 |  |
| 3-7 | 6.3Computations of Derivatives | Introduction of alternative notation for the derivative of a function Introduction of the derivative rule for simple powers * Students use the derivative rule to calculate the derivatives of functions with integer exponents
 |  |
| 6.4Properties of Derivatives | Discussion on the features of the derivative* Is it a function? (Definition of a function readdressed from Topic 1 or briefly given if Topic 1 has not been done)

Recognition and use of the linearity of the derivative* Students establish, by working through appropriate examples, rules of differentiation for simple powers:

 If  If  |
| 3-8 | 6.5Applications of Derivatives | Using derivatives of polynomials and other linear combinations of power functions to determine the equation of a tangent to a curve at a pointUnderstanding the following * Review sign diagrams
* Develop the concepts of displacement and velocity (use of position versus time graphs to describe motion where velocity equates to the slope of the tangent at any point on the graph)
* Rates of change - increasing and decreasing - use of sign diagram to determine intervals in which the function is increasing and decreasing
* Maxima and minima - local and global
* Stationary points and end points

Use of the above to find * Velocity from displacement (first derivative of displacement)
* Object is at rest when velocity is zero
* When an object changes direction i.e. when velocity equals zero and there is a change of sign (sign diagram required)

Optimisation* Examine various optimisation problems (only in consideration of simple polynomials and other linear combinations of power functions) in relation to real life situations such as cost minimisation, optimisation of dimensions of 3D objects, and water use.
 | **INVESTIGATION** Modelling With DerivativesCake Tin Optimisation |
| 3-9 |  |
| 3-10 |  | **Revision and SAT 2****Investigation submission** | **SAT 2 Part 1** – No calculatorFirst principles and derivatives of polynomials**SAT 2 Part 2** – Calculator permitted6.4 and 6.5 |

**Topic 5 – Growth and Decay (4 weeks)**

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| **Term Week** | **Subtopic** | **Concepts and Content**Technology is incorporated into all aspects of this topic as appropriate | **Assessment Task** |
| 4-1 | 5.1 Indices and Index Laws | Indices* Review indices and index laws including negatives and fractional
* Algebraic application to all laws including simplification using positive, negative and fractional indices
* Conversions from radical to fractional indices

Surds* Definition of rational and irrational numbers
* Operations with surds and fractional indices (rational indices)
* Discussion on the real number system and its inclusion of irrationals
 |  |
| 4-2 | 5.2Exponential Functions | Exponentials* Exponential functions - their algebraic properties and uses
* Behaviour of exponential functions
* Technology will be used to explore the qualitative features of the graph of , its translations and and dilation
* Discussion on characteristics such as asymptotes, intercepts and behaviour as
* \*Use of real life situations to determine variables in the contexts such as bacteria growth, radioactive decay, half-life, population models and compounding interest. Technology is used to support interpretation of situations.
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| 4-3 | 5.3Logarithmic Functions | Definition of a logarithm, initially base 10* Rules, initially base 10

Definition of a logarithm other bases* Application of rules with other bases

Solving of logarithmic equations (base 10)Solving exponential equations using logarithms (base 10) threaded back to exponentials dot point 5\* (from subtopic 5.2) |  |

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| 4-4 |  | **Revision and SAT 3** | **SAT 3**Entire topicCalculator permitted  |
| 4-5 |  | **EXAMINATION REVISION** |  |
| 4-6 |  | **YEAR 11 EXAMS** |  |